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3-ACT MATH Tasks: Authentic Engagement with Mathematical Ideas

By Zachary Champagne and Jennifer M. Suh

3-ACT MATH Tasks are built upon this foundational idea: *Students are more engaged in mathematics when they are authentically invested in the task.* As will be detailed in this paper, this investment is far beyond that which students generally experience with a traditional "real-world" task.

The basic structure of a 3-ACT MATH Task is based on storytelling. Books and movies often tell their story in three parts: conflict is introduced, characters look for clues and resources, conflict is resolved. Mathematics educators (Meyer, 2011) have noticed that this framework for storytelling maps nicely onto high quality math tasks. These tasks begin with "conflict": an intriguing image or video that is intended to pique the student's interest. From there students are encouraged to pursue questions that they have based on the video, consider what information they need to find the answers to those questions, and finally ("conflict resolved") use mathematics to answer the question.

What Happens in ACT 1?

In ACT 1 of these tasks, the teacher presents a striking visual (image or video vignette) that is intriguing and engaging—intended to draw students into the problem. Important to ACT 1 is simplicity—but simplicity that requires students to want to know and to do more. Dan Meyer (2011) says, "Your first act should impose as few demands on the students as possible—either of language or of math. It should ask for little and offer a lot."

Source: enVision® Mathematics *©2020, Grade K, (Charles et al., 2020)*

Figure 1. A boy first grabs a handful of grapes.

Most of the 3-ACT MATH Tasks in *enVision* have video vignettes for ACT 1. As an example, consider the following screen shots from a Kindergarten 3-ACT MATH Task.

As students watch the boy's fingers clutching at grapes in the bowl, they will likely wonder, "How many grapes did he grab?" Next a girl enters the scene, and she also grabs a handful of grapes.

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Figure 2. A girl then grabs a larger handful of grapes.

After students watch the video (or view an image) in ACT 1, they are asked two simple, yet crucial questions: *What do you notice? What do you wonder?* The class brainstorms a multitude of wonderings that the class community has. Students are encouraged to discuss all the noticings and wonderings from their classmates, and through discussion as a group (including the teacher), the class settles on a *main question* to pursue. In this example, the main questions would be, "How many grapes did they each grab?" and "Who grabbed more grapes?"

In another critical component of ACT 1, students estimate, or predict, the answer to the main question. This estimate is much more than just a guess. They watch the video again closely and discuss what would be a reasonable answer to the main question. In this example, students use what they know about the size of their own hands and watch to determine a reasonable estimate for how many grapes the boy and the girl each grab.

What Happens in ACT 2?

In ACT 2, students begin to consider what information they would need to be able to answer the main question. The teacher's role is to provide that information. The key here is that students are now looking for the information they need to solve the problem—one of the pillars of a powerful problem-solving task.

In this example, the teacher presents these two follow-up images to offer the information students need.

Source: enVision® Mathematics *©2020, Grade K (Charles et al., 2020)*

Figure 3.

Source: enVision® Mathematics *©2020, Grade K (Charles et al., 2020)*

Figure 4. The boy and girl showing the grapes they grabbed in consecutive images.

From these two images, students are able to count the number of grapes that each child grabbed and then choose ways that make sense to them to determine who had more grapes. If needed, another image can be provided to guide this conversation.

Source: enVision® Mathematics *©2020, Grade K (Charles et al., 2020)*

Figure 5. The girl and boy comparing their grapes in one image.

What Happens in ACT 3?

In ACT 3, students get the payoff for their hard work: *The answer to the main question is revealed*. The word "revealed" is important here. Remember, students have a vested interest in this problem. They deserve a resolution that is powerful and much bigger than "The girl has more grapes." In this task, the video reveals each handful, and then counts the number of grapes in each hand, highlighting one at a time.

Source: enVision® Mathematics *©2020, Grade K (Charles et al., 2020)*

Figure 6.

Source: enVision® Mathematics *©2020, Grade K (Charles et al., 2020)*

Figure 7. The handfuls of grapes, highlighted with the final count.

The payoff will be noticeable. Once you do a 3-ACT MATH Task with your students, you'll find that in ACT 3, there is almost always a cheer. Students are invariably excited to see an authentic reveal of the solution to a problem that they chose to solve, and solved in a way that made sense to them.

Oftentimes, after ACT 3, "sequels are provided for students to explore. These sequels push further questions and ideas that are related to the mathematics involved in the task."

Important Note: In this particular task, students are provided in ACT 2 with the exact information that is necessary to determine the solution to the main question. Some of these tasks have more ambiguity; at times, the information given isn't exactly clear, and estimations or approximations are necessary. There are also times when the solution in ACT 3 brings a surprise or twist. This is intentional, and helps students understand the interaction between mathematics, modeling, and the real world.

Part 2: Why 3-ACT MATH Tasks?

3-ACT MATH Tasks engage, involve, and challenge students at all levels. A well-constructed 3-ACT MATH Task allows each and every student in your classroom, on some level, to access the mathematical ideas inherent in the work. It also provides paths for students to pursue the question(s) at higher levels. For example, in the task above, most Kindergarten students can see that the girl in this video has "more" grapes. They may even be able to do so without counting. However, students could pursue a variety of follow-up questions, including but not limited to these:

- *How many more grapes does she have than he does?*
- *How many grapes does she need to eat so that they have the same amount?*
- *What do you notice about the green grapes? The purple grapes?*
- *If the boy got one more green grape, would he have more than the girl?*

A second, yet equally important, component of these tasks is that students have the chance to ask authentic questions. When teachers ask, "What do you notice? What do you wonder?," they are genuinely interested in hearing how students respond, because there are no expected answers; no one knows what the students might in fact be wondering about. As teachers show that they are invested in the students' expressed ideas, the students themselves become authentic learners; both sides are authentically engaged in the conversation about the situation.

The estimation skills that students acquire through 3-ACT MATH Tasks are remarkable. As students have continued conversations around estimates that are too high and estimates that are too low, they become better estimators as they use and think carefully about the information that they are given in ACT 1. They learn to do less guessing and to think about the situation and various factors that may influence the solutions.

When students engage with 3-ACT MATH Tasks, they are building agency and engaging in tasks where they have choice and input on what ideas are pursued. The impact of choice and self-determination on learning is well documented in the research and literature. In fact, providing students with choice and/or more autonomy has been shown to:

- Increase attendance and increase scores on a national test of basic skills than those in conventional classrooms. (de Charms, 1972)
- Help students develop more sophisticated reasoning skills without falling behind on basic conceptual tasks. (Cobb, et al. 1991; Yackel, Cobb, and Wood, 1991)

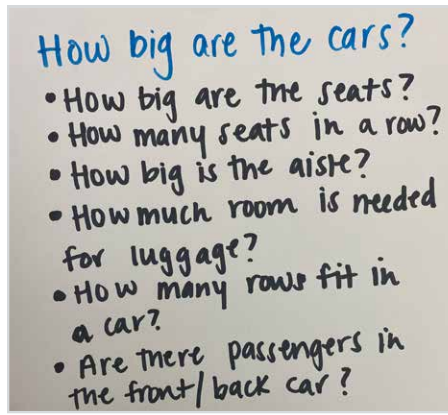
Part 3: 3-ACT MATH Tasks and the Mathematical Practices and the Progression to Mathematical Modeling

These 3-ACT MATH Tasks can support teachers and students in developing the mathematical practices as presented by state standards. One of the most significant mathematical practices supported by 3-ACT MATH is the *Standards for Mathematical Practice MP4: Model with mathematics*. The video vignettes provide students a way to visualize the problem situation and relate the problems to real-life scenarios. In elementary grades, this may start with students writing an equation to describe a situation, then move towards the modeling process—making assumptions, building a solution, then revising their thinking, by reflecting on the real-world situation and improving on their mathematical solution or model. This leads in a seamless progression to more formalized mathematical modeling tasks in the upper grades.

As students pose the problem in ACT I, they are working to *make sense of problems and persevere in solving them* (MP1). As they talk about the task, students make sense of the problem by asking questions, promoting inquiry-based thinking. In ACT 1 of a fifth grade 3-ACT MATH Task featured below, students watch a video vignette of a train moving by with multiple cars. After prompting students to generate their noticings and wonderings, the teacher could pose one of their questions to engage the class, such as "How many seats are on this train?" Looking at the photo, students will have to make some assumptions. They may have to consider how many seats are in each row and the arrangement of seats on the train.

Source: enVision® Mathematics *©2020, Grade 5 (Charles et al., 2020)*

Figure 8.



Source: enVision® Mathematics *©2020, Grade 5 (Charles et al., 2020)*

Figure 9. In ACT 1, viewing an image generates noticings and wonderings.

In ACT 2, teachers can focus on having students *reason abstractly and quantitatively* (MP2) as they make assumptions and consider what information they would need to answer that main question. Students make educated estimates that allow teachers to assess their number sense and quantitative reasoning skills.

Source: enVision® Mathematics *©2020, Grade 5 (Charles et al., 2020)*

Figure 10. In ACT 2, students are provided data and/or more information to mathematize the problem.

Based on the assumptions, students will determine pathways to solve the problem. For example, they may write a number sentence or build mathematical equations. The teacher may prompt students to attend to precision (MP6) as they give carefully formulated explanations to each other. A student thinking aloud might sound like this: "I am going to assume that there are 2 seats on both sides of a row and that there are 7 rows in each passenger car. That would mean 7 rows x 4 sets = 28 seats in each car and 28 seats x 11 cars = 308 total seats."

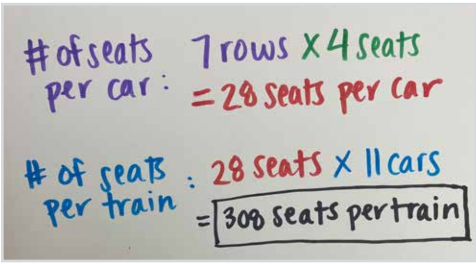


Figure 11. An example of a solution derived through a multi-step problem.

In this example, students would also *look for and make use of structure* (MP7) in their computational work. For example, some students might use multiplication to solve the problem, while others might use repeated addition. As students discuss and *critique the reasoning of others* (MP3), they may notice that there are patterns in their thinking. They may notice that some of them are approaching it as a multi-step problem, as shown above in Figure 8.

While comparing methods and considering the many different configurations, students will notice that the variables important to this problem are considering how many seats in each row, how many rows in each car, and how many cars in total. Students may use algebraic structures, methods, and patterns as they *look for and express regularity in repeated reasoning* (MP8).

The sample work below shows how some students may use pictures as a tool to represent and guide their thinking. Others might use counting cubes to build the rows of seats in the car. This use of tools supports *using appropriate tools strategically* (MP5) to represent their thinking. In addition, students may use symbols to represent the variables: a triangle to represent the number of seats, a cloud to represent the number of rows, and a box to represent the number of cars. Researchers have emphasized that in algebra problem-solving situations, it is not merely getting the answer that counts; just as important is emphasizing how these situations are about relationships among solving methods, and finding ways to represent these relationships and methods with equations that are as generalizable as possible (Kieren, 2007; Boaler and Humphreys, 2005).

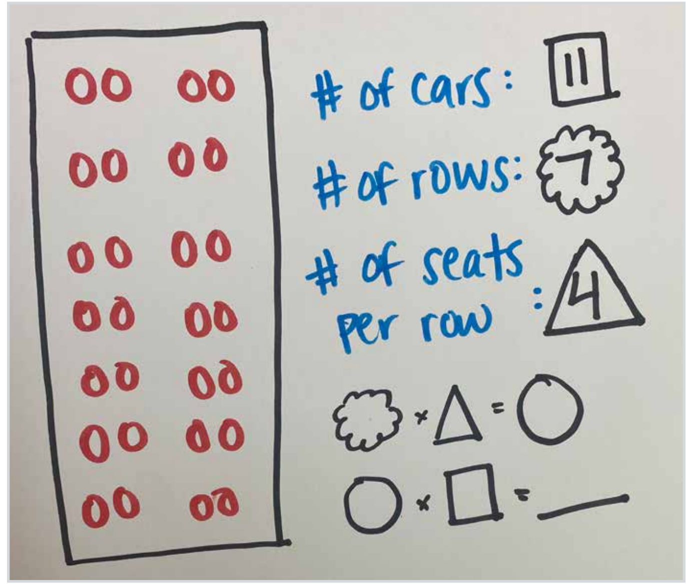


Figure 12. Visual representation of one of the passenger cars, and using algebraic thinking.

In other words, the algebraic equation would allow for students to create a generalizable model. Another way to represent a generalizable model for calculating total number of seats is: $t = s \times r \times c$, where s represents the number of seats; r represents the number of rows, and *c* represents the number of passenger cars. Although some students may be at a more concrete stage with numeric equations, others might verbalize, "I will take the number of seats and multiply it by the number of rows to find out how many in each car. Then I can take that number and multiply it by the number of cars."

The most exciting part is providing students with the opportunity to engage in a healthy mathematical argumentation. Based on different assumptions, students may have different answers. However, an important aspect of these tasks is that students are given the opportunity to *construct viable arguments and critique the reasoning of others* (MP3). In so doing, they learn to value different perspectives and appreciate the variety of ways people find to solve problems.

When more information is revealed in the final ACT, in a form of a photo or a video vignette, students are given an opportunity to compare their assumptions and the reasonableness of their solutions. In a culminating image of this 3-ACT MATH Task, students get to see inside of the passenger car. This is where they may revise their thinking as they realize, "Wow, there is a dining section! And the car is bigger than I thought. There are 14 rows!" They may come to realize that they have an equation that will work for any configuration (e.g., *t* = *s x r x c*. In this case, *t* = 4 x 14 x 11, or *t* = 616 total seats at full capacity).

Source: enVision® Mathematics *©2020, Grade 5 (Charles et al., 2020)*

Figure 13. The final reveal allows students to compare their assumptions and the reasonableness of their solutions.

Conclusion

An overarching goal of including 3-ACT MATH Tasks in classrooms is to engage students in being both problem posers and problem solvers as they determine solutions to authentic questions. They are put in the driver's seat as they both pose the questions and determine the solutions to those problems. This sets our students up for success as they become people who can take problems from the real world and mathematize those situations, making wise decisions using the most important tool, mathematics! Early introduction to this idea of modeling allows for a natural progression to the middle and high school standards for mathematical modeling.

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