

Using Operations to Solve Problems



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Online Resources

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Introduction to Math Navigator

Dear Parent/Guardian,

_____ has been selected to participate in Math Navigator! Math Navigator is one of the ways that our school is working to help all students succeed in mathematics. The program gives students the additional time and instruction they need to improve their performance in this important subject.

Your child will be participating in the *Using Operations to Solve Problems* module. The main goal of this module is to help students solve one-step and two-step word problems. The module exposes students to a variety of problem situations involving addition, subtraction, multiplication, and/or division. Students will learn to represent problem situations with diagrams or other representations, to identify the unknown quantity, and to write an equation with a variable to stand for the unknown quantity. Students will learn to check the reasonableness of their answer by making an estimate first. They will work with the order of operations to solve problems involving more than one operation. At the end of the module, students will use information from charts, tables, pictographs, and bar graphs to solve one-step and two-step word problems.

There are a variety of materials students will use with this module: one of them is a set of Study Cards. These cards include mathematical ideas for students to master, game cards, and blank cards that students can customize with concepts that they need to work on. Students are encouraged to use these cards during the lessons, as well as during free time and at home. Please encourage your child to share them with you.

The more enthusiastic you can be about Math Navigator, the more it will help your child. Ask questions each day about what your child learned and how the Math Navigator class was different from your child's regular math class. It is important for you to acknowledge what your child has accomplished both on a day-to-day basis and after completing the Math Navigator module.

We are excited about using Math Navigator with students. Learn more about this special program and how it works by reading the short description that follows. If you have any questions about the program, please do not hesitate to contact us here at school.

How Math Navigator Works

Structure of a Module

Each module contains 20 days of 30- or 45-minute lessons, including a pre-test and post-test. During the 20 days, students have two or three checkpoint lessons that assess their understanding of the concepts in the module.

Frequent Skills Practice

Most lessons include a Show Me session in which students practice and reinforce skills. It is also a time for students to learn strategies and techniques that make computation easier.

Emphasis on Understanding

The lessons are carefully designed to uncover mistakes that result from students misunderstanding something. We call such mistakes *misconceptions*. Misconceptions need to be corrected because they can interfere with new learning. Math Navigator modules do not attempt to reteach everything that students have learned about a topic. Instead, they help students understand the mathematics of the procedures and concepts that they have already learned so that they can correct the misconceptions that are getting in the way of their progress.

Learning to Think Mathematically

Lessons are structured to teach students to think like mathematicians. Students will learn how to ask themselves questions before beginning a problem; to use diagrams, tables, and other methods of representing problems; and to estimate as a way of determining whether their answers are reasonable. Most importantly, they will come to see that mistakes are opportunities for learning, rather than something to hide.

Misconceptions and Errors

A2	Does not connect the information in the representation to the quantities in the problem
DP1	Thinks that icons on pictographs, boxes on bar graphs, or intervals always represent one of something
DP5	Is unable to follow the rows or columns of a table or graph accurately and thus misreads the table or graph
DP8	Does not connect the relative lengths of the bars in a bar graph to quantities in the situation
E1	Fails to apply or misapplies order of operations principles
E19	Thinks a variable represents an object rather than a number
G6	Does not know how to represent word problems as diagrams or models
G8	Does not recognize the implied data in a word problem—unnamed quantities that must be calculated in multistep problems
NL7	Fails to interpret the scale on a graph or number line
O1	Does not use the correct operation
O3	Does not recognize addition situations
O4	Does not recognize subtraction situations
O5	Does not recognize multiplication situations
O6	Does not recognize division situations
O17	Estimates incorrectly
O22	Fails to link addition and subtraction or multiplication and division as inverse operations
O34	Does not complete all the steps of a multistep problem

A2 Does not connect the information in the representation to the quantities in the problem

The student does not understand the problem and cannot connect the information in the problem to the question being asked.

example

Josh can read 40 pages in an hour. If he reads for an hour each day, how many pages will he read in 7 days?

Which table can be used to solve this problem?

The student chooses:

Days	1	2	3	4	5	6	7
Pages	40	40	40	40	40	40	40

Consider the following table.

Cost of School Lunches Over the Years

Year	Cost of School Lunch
2003	\$1.80
2004	\$1.85
2005	\$1.85
2006	\$1.90
2007	\$2.00

Create three questions that can be answered by looking at the information in the table.

Question 1:

How many students went to school in 2003?

DP1 Thinks that icons on pictographs, boxes on bar graphs, or intervals always represent one of something

The student does not realize that icons on a graph do not have to represent just one of something. The student does not know how to read legends or scales on the graph.

example

How many students like chocolate ice cream?

Our Favorite Ice Cream Flavors
Third-Grade Students at Brightside Elementary

Legend:
○ = 10 students

Flavor	Number of Icons	Number of Students
Chocolate	6	60
Vanilla	3	30
Strawberry	4	40
Chocolate Chip	10	100

6 students like chocolate ice cream.

DP5 Is unable to follow the rows or columns of a table or graph accurately and thus misreads the table or graph

example

Use this table to find out how much 3 pencils cost.

Cost of Pencils

Number of Pencils	Cost
1	20¢
2	40¢
3	60¢

3 pencils cost 20 cents.

DP8 Does not connect the relative lengths of the bars in a bar graph to quantities in the situation

The student may be able to look at a graph and say which data category has more based on the lengths of the bars but cannot tell you “how many” the bar represents or “how many more” one bar represents as compared to another.

example

Which activity do the most students prefer on a weekend?

The activity most students prefer is listening to music.

How many students prefer to listen to music on the weekend?

3 students prefer to listen to music.

How many more students prefer to listen to music than to do homework?

More students prefer to listen to music.

Preferred Weekend Activities of Students in Mrs. Chi’s Class

Activity	Number of Students
Visit with Friends	10
Homework	5
Listen to Music	15

E1 Fails to apply or misapplies order of operations principles

The student solves problems from left to right no matter what the operations are.

example

$$20 + 6 \cdot 3 = 78$$

$$(2 \cdot 9) + 22 - 20 - (8 \cdot 2) = 24$$

example

Work with a partner.

1. Jong handed in the following homework. For each problem, correct any wrong answers and explain what mistake(s) he made.

10. When you add 7 to an unknown number and then multiply by 4, it equals 40. What is the unknown number?

$x + 7 \cdot 4 = 40$ $(x+7) \cdot 4 = 40$ $x+7=10$
 $x + 28 = 40$ $\frac{4}{4} \frac{4}{4}$ $\frac{-7}{-7}$
 $x + 28 - 28 = 40 - 28$ $x = 33$
 $x = 12$

E19 Thinks a variable represents an object rather than a number

example

“If there are d days in w weeks, then $w = 7d$ because a week equals 7 days.” This interpretation is incorrect because w and d are identified as the “object” week and day, rather than as the numbers of weeks and days. A correct equation would be $d = 7w$, because we would have to multiply the number of weeks by 7 to get the number of days.

G6 Does not know how to represent word problems as diagrams or models

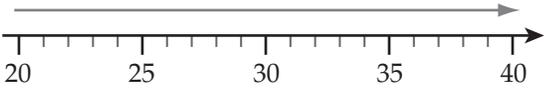
The student does not know how to represent a word problem in a diagram or model. Since the diagram illustrates an understanding of the problem, the student is not understanding the relationships between the quantities.

example

Mr. Hakim buys 40 oranges. He shares the oranges equally among 8 people. How many oranges does each person get?

Draw a diagram that can be used to solve this problem.

This diagram represents the situation:



G8 Does not recognize the implied data in a word problem—unnamed quantities that must be calculated in multistep problems

example

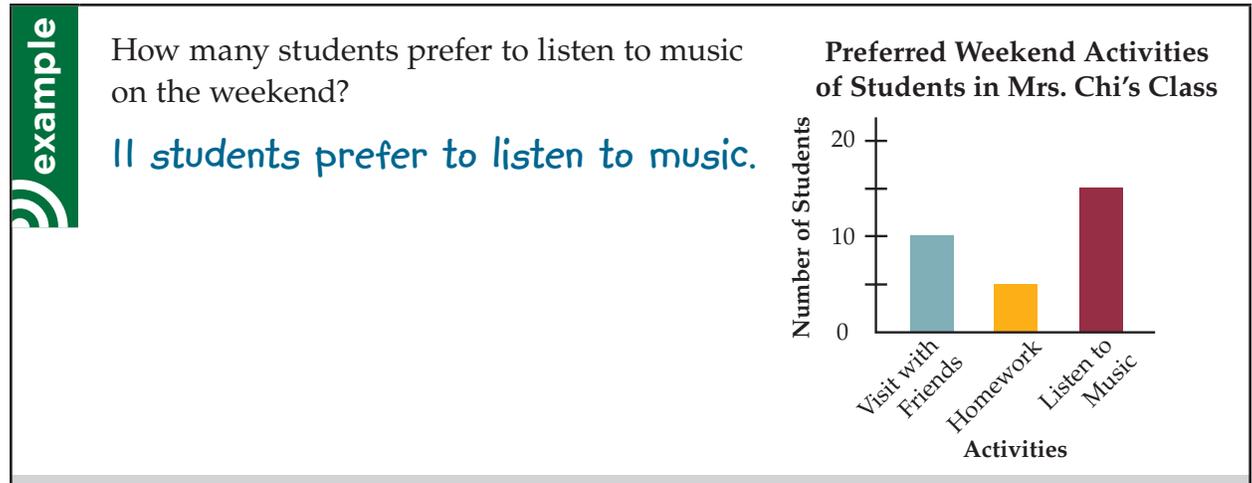
Amir wants to swim 200 laps at swim practice today. He decides to practice his backstroke for 15 minutes and then switch to freestyle. When he does the backstroke, he swims 5 laps per minute.

How many laps will he need to swim freestyle to reach his goal of 200 laps?

Amir will need to swim 195 laps.

NL7 Fails to interpret the scale on a graph or number line

When asked to read or interpret a graph, the student does not read the scale correctly.



O1 Does not use the correct operation

The student does not choose the correct operation needed to solve the problem—he adds instead of subtracting, multiplies instead of dividing, and so on.

example

Three students each have 9 apples. How many do they have altogether?

$3 + 9 = 12$

The temperature was 47° at 8:00 AM, which was 12° cooler than it is now. What is the temperature now?

The temperature now is 35° . $47 - 12 = 35$

There are 12 treats to be shared among 4 dogs. How many treats does each dog get?

$12 - 4 = 8$. Each dog gets 8 treats.

Tom and Megan have 24 books altogether. Megan has 12 books. How many books does Tom have?

$24 + 12 = 36$. Tom has 36 books.

O3 Does not recognize addition situations

The student overspecializes during the learning process so that he recognizes some situations as addition but fails to classify other addition situations appropriately.

example

The temperature was 47° at 8:00 AM, which was 12° cooler than it is now. What is the temperature now?

The temperature now is 35° . $47 - 12 = 35$

O4 Does not recognize subtraction situations

The student overspecializes during the learning process so that she recognizes some subtraction situations as subtraction but fails to classify other subtraction situations appropriately.

example

If there are 7 birds in a bush and 3 fly away, how many are left?

$7 - 3 = 4$. 4 birds are left.

Tom and Megan have 24 books altogether. Megan has 12 books. How many books does Tom have?

$24 + 12 = 36$. Tom has 36 books.

O5 Does not recognize multiplication situations

The student knows how to multiply but does not know when to multiply (other than because she was told to do so, or because the computation was written as a multiplication problem). The student cannot explain why she should multiply.

example

Three students each have 9 apples. How many do they have altogether?

$3 + 9 = 12$

O6 Does not recognize division situations

The student knows how to divide but does not know when to divide (other than because he was told to do so, or because the computation was written as a division problem). The student cannot explain why he should divide or connect division to actions with materials.

example

Three students share 9 apples. How many did each one get?

$$3 + 9 = 12$$

O17 Estimates incorrectly

example

What is a good estimate for the solution to the following problem?
(Do not solve for the exact answer.)

Angela cycles 36 miles on Saturday and 48 miles on Sunday. How far does she cycle both days in total?

Angela rides about 70 miles in total.

O22 Fails to link addition and subtraction or multiplication and division as inverse operations

The student sees addition and subtraction as discrete and separate operations, as well as multiplication and division. Her conception of the operations does not include the fact that they are linked as inverse operations. Student knows procedures for subtracting and dividing but has no idea how to check the reasonableness of her answers.

example

$$8 + 9 = 17$$

$$17 - 8 = 11$$

$$6 \times 7 = 42$$

$$42 \div 7 = 8$$

O34 Does not complete all the steps of a multistep problem

The student reads only part of the problem—he assumes all problems involve only one step and thus leaves out some of the information in the problem.



Maria went to the beach. On Monday, she collected 15 shells. On Tuesday, she collected 26 more. On Wednesday, she added 12 shells to her collection. How many shells did she collect in all?

$26 + 12 = 38$. She has 38 shells.

Class Profile Instructions

About the Class Profile

Completing an analysis of student work gives you a clear picture of the strategies an individual student is applying to a particular problem or topic in mathematics. Such an analysis is even more powerful when it is applied to the Math Navigator class as a whole.

The Class Profile gives you both. By reading the Class Profile across a row, you can see where each student stands at any point in time. Reading down the columns allows you to see the strengths and needs of the entire class at a glance. By reviewing the Class Profile, you will be able to make decisions that target appropriate instruction to individuals, small groups, and the whole Math Navigator class.

The first pages of the Class Profile provide assessment items related to the content of the module. The last page is based on the mathematical practices from the Common Core State Standards for Mathematics.¹ On this page, record evidence of students using these practices.

Recording Data on the Class Profile

When you see—either through discussion, analysis of student work, or direct observation—that a student understands a concept, still has a misconception, or engages in a mathematical practice, make a note on your Class Profile. As the student's understanding increases, update the Class Profile.

Using the Class Profile

Review the Class Profile periodically during the lesson to help you decide which topics would be most beneficial for your students to focus on during the class discussion. Address topics that most of the students in the Math Navigator group need to learn during the show me, work time, or probing for understanding parts of the lesson. Address topics that only some students are struggling with during partner work or in conferences. If only one or two students need help with a topic, address the topic in an individual conference.

Give a copy of the completed Class Profile to each student's classroom teacher at the end of the module.

¹Common Core State Standards Initiative. 2010. "Common Core State Standards for Mathematics": 6–8. Accessed July 1, 2011. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

CLASS PROFILE (1 OF 3)

		Concepts								
10 9 8 7 6 5 4 3 2 1	Student Name	<i>C1: Recognizes addition situations</i>								
		<i>C2: Recognizes subtraction situations</i>								
		<i>C3: Recognizes multiplication situations</i>								
		<i>C4: Recognizes division situations</i>								
		<i>C5: Recognizes the hidden question in a two-step problem</i>								
		<i>C6: Writes equations with a letter standing for the unknown quantity to represent problem situations</i>								
		<i>C7: Uses visual representations to represent problem situations</i>								
		<i>C8: Interprets data in tables, charts, and graphs</i>								
		<i>C9: Understands division as an unknown-factor problem</i>								
				Observed Errors						

CLASS PROFILE (2 OF 3)

Concepts and Procedures	
1	Student Name
2	3
3	4
4	5
5	6
6	7
7	8
8	9
9	10
10	Observed Errors

C10: Understands subtraction as an unknown-addend problem

P1: Solves one-step word problems

P2: Solves two-step word problems using the four operations

P3: Uses mental computation and estimation strategies to check the reasonableness of answers

P4: Uses the order of operations

P5: Extracts information given in tables, charts, and graphs accurately

P6: Solves "how many more" and "how many less" problems using information presented in scaled bar graphs

Mathematical Practice Standards	
<p>MP1: Make sense of problems and persevere in solving them.</p> <p>MP2: Reason abstractly and quantitatively.</p> <p>MP3: Construct viable arguments and critique the reasoning of others.</p> <p>MP4: Model with mathematics.</p>	<p>MP5: Use appropriate tools strategically.</p> <p>MP6: Attend to precision.</p> <p>MP7: Look for and make use of structure.</p> <p>MP8: Look for and express regularity in repeated reasoning.</p>
Student Name	Observations
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



A Complete Solution to a Word Problem

includes all of the following ...



A written estimate



All work that you do



An equation (even if you solved it using column form)



A diagram, number line, table, or other representation



The answer to the question in a complete sentence



What to Do If You Get Stuck



Look at past work times



Look at the charts that are posted



Model the problem using counters or other materials



Sketch a diagram or other representation



Change the numbers to make the problem simpler



Write what you do know



Write down questions to ask later



Check other resources