

Using Expressions and Equations to Solve Problems



AMERICA'S
CHOICE®

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Online Resources

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Introduction to Math Navigator

Dear Parent/Guardian,

_____ has been selected to participate in Math Navigator! Math Navigator is one of the ways that our school is working to help all students succeed in mathematics. The program gives students the additional time and instruction they need to improve their performance in this important subject.

Your child will be participating in the *Using Expressions and Equations to Solve Problems* module. The main goal of this module is to give students practice with solving real-life, multistep mathematical problems using a variety of strategies. Students will represent relationships using tables, graphs, equations, and scale models. They will practice interpreting word descriptions of problem situations as well as identifying and describing the relationships between the key variables in a problem situation.

The lessons will help students develop independence in solving problems. The beginning lessons contain more scaffolding, while lessons later in the unit are more open-ended and require students to utilize strategies that they learned in earlier lessons.

There are a variety of materials students will use with this module: one of them is a set of Study Cards. These cards include mathematical ideas for students to master, and blank cards that students can customize with concepts that they need to work on. Students are encouraged to use these cards during the lessons, as well as during free time and at home. Please encourage your child to share them with you.

The more enthusiastic you can be about Math Navigator, the more it will help your child. Ask questions each day about what your child learned and how the Math Navigator class was different from your child's regular math class. It is important for you to acknowledge what your child has accomplished both on a day-to-day basis and after completing the Math Navigator module.

We are excited about using Math Navigator with students. Learn more about this special program and how it works by reading the short description that follows. If you have any questions about the program, please do not hesitate to contact us here at school.

How Math Navigator Works

Structure of a Module

Each module contains 20 days of 30- or 45-minute lessons, including a pre-test and post-test. During the 20 days, students have two or three checkpoint lessons that assess their understanding of the concepts in the module.

Frequent Skills Practice

Most lessons include a Show Me session in which students practice and reinforce skills. It is also a time for students to learn strategies and techniques that make computation easier.

Emphasis on Understanding

The lessons are carefully designed to uncover mistakes that result from students misunderstanding something. We call such mistakes *misconceptions*. Misconceptions need to be corrected because they can interfere with new learning. Math Navigator modules do not attempt to reteach everything that students have learned about a topic. Instead, they help students understand the mathematics of the procedures and concepts that they have already learned so that they can correct the misconceptions that are getting in the way of their progress.

Learning to Think Mathematically

Lessons are structured to teach students to think like mathematicians. Students will learn how to ask themselves questions before beginning a problem; to use diagrams, tables, and other methods of representing problems; and to estimate as a way of determining whether their answers are reasonable. Most importantly, they will come to see that mistakes are opportunities for learning, rather than something to hide.

Misconceptions and Errors

A5	Thinks that a graph is just a picture of a situation
A12	Is unable to transfer between verbal, tabular, graphical, and/or algebraic representations of linear functions
A25	Chooses inappropriate units for the graph axes
E9	Misapplies the properties of equality
F6	Misapplies rules for converting from fractions to decimals or percents
G6	Does not know how to represent word problems as diagrams or models
G8	Does not recognize the implied data in a word problem—unnamed quantities that must be calculated in multistep problems
M7	Confuses the formulas for area and perimeter
M18	Does not understand or ignores unit conversion
M19	Does not understand the relationship between the quantities in the problem
PCT2	Does not understand that a percentage is a fraction out of 100
O1	Does not use the correct operation
O17	Estimates incorrectly
O34	Does not complete all the steps of a multistep problem
RP1	Uses additive reasoning for ratios, rates, or proportional relationships
RP6	Uses intuitive methods to solve ratio and proportion problems (e.g., adding or halving), but cannot generalize these methods to other ratio/proportion problems

A5 Thinks that a graph is just a picture of a situation

example Sketch a graph that best shows the speed of a bouncing ball over time.

A12 Is unable to transfer between verbal, tabular, graphical, and algebraic representations of linear functions

example Sketch a graph that represents the relationship shown in this table.

x	-1	0	1	2	3
y	-1	1	3	5	7

Response:

A25 Chooses inappropriate units for graph axes

example Patti ran 2.5 kilometers in 30 minutes. Sketch a graph showing her distance over time.

E9 Misapplies the properties of equality

Confuses negative signs when adding and subtracting terms

example

$$2x + 12 = x$$

$$x = 12$$

$$2x + 3 = x + 4$$

$$x = 7$$

F6 Misapplies rules for converting from fractions to decimals or percents

example

$$76\% = 0.0076$$

$$\frac{1}{5} = 0.02$$

G6 Does not know how to represent word problems as diagrams or models

The student does not know how to represent a word problem in a diagram or model. Since the diagram illustrates an understanding of the problem, the student is not understanding the relationships between the quantities.

example

Mr. Hakim bought 8 oranges. The cost of the oranges was \$4.40. How much did he pay per orange?

Draw a diagram that can be used to solve this problem.

This diagram represents the situation:

G8 Does not recognize the implied data in a word problem—unnamed quantities that must be calculated in multistep problems

example

Amir wants to swim 200 laps at swim practice today. He decides to practice his backstroke for 15 minutes and then switch to freestyle. When he does the backstroke, he swims 5 laps per minute.

How many laps will he need to swim freestyle to reach his goal of 200 laps?

Amir will need to swim 195 laps.

M7 Confuses the formulas for area and perimeter

The student interprets all “fence” problems as perimeter, even if the problem talks about the size of the garden that the fence encloses, or the student interprets all “wall painting” problems as area, even if the problem talks about the length of a striped border that is painted around the room.

example

Find the area of a rectangle with dimensions of 12 cm × 4 cm.

$$12 + 4 + 12 + 4 = 32 \text{ cm}$$

example

Find the perimeter of a rectangle with dimensions of 8 inches × 7 inches.

$$8 \times 7 = 56 \text{ inches}$$

M18 Does not understand or ignores unit conversion

The student does not convert correctly between different units of measure—linear, capacity, time, and so on.

example

Malaya went to the mall to shop. After 90 minutes, she decided to see a movie at the cinema in the mall. The movie took 2 hours and 15 minutes. Then she left for home.

How long was Malaya at the mall?

Malaya was at the mall for 3 hours and 5 minutes.

M19 Does not understand the relationship between the quantities in the problem

In order to solve a word problem, the student needs to understand the relationships between the quantities—how one relates to another and what they mean in the context of the problem.

example

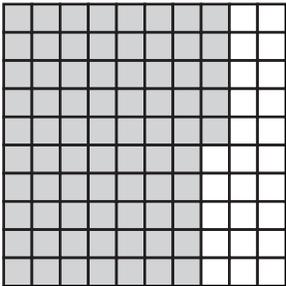
Mrs. Chi bought a loaf of bread and 4 apples for \$4.39. If the bread cost \$2.59, what did she pay for each apple?

Mrs. Chi paid \$1.80 for each apple.

PCT2 Does not understand that percents refer to hundredths

example

What percent of the whole square is shaded?



750%

Change this decimal to a percent.

0.253%

O1 Does not use the correct operation

The student does not choose the correct operation needed to solve the problem—he adds instead of subtracting, multiplies instead of dividing, and so on.

example

Three students each have 9 apples. How many do apples have altogether?

$$3 + 9 = 12$$

The temperature was 47°F at 8 AM, which was 12°F cooler than it is now. What is the temperature now?

The temperature now is 35°F. $47 - 12 = 35$

There are 12 treats to be shared among 4 dogs. How many treats does each dog get?

$12 - 4 = 8$; each dog gets 8 treats.

Tom and Megan have 24 books altogether. Megan has 12 books. How many books does Tom have?

$24 + 12 = 36$; Tom has 36 books.

O17 Estimates incorrectly**example**

What is a good estimate for the solution to the following problem? (Do not solve for the exact answer.)

Angela cycles 11.6 miles at a steady speed of 20.4 miles per hour. How long does she ride?

Angela rides for 0.40 hour.

O34 Does not complete all the steps of a multistep problem

The student reads only part of the problem—he assumes all problems involve only one step and thus leaves out some of the information in the problem.

example

Maria went to the beach. On Monday, she collected 15 shells. On Tuesday, she collected 26 more. On Wednesday, she added 12 shells to her collection. How many shells did she collect in all?

$26 + 12 = 38$; She has 38 shells.

RP1 Uses additive reasoning for ratios, rates, or proportional relationships

example

There are 25 first class seats and 50 economy class seats on an airplane. The airline has a smaller plane with 20 first class and the same ratio of first class to economy seats. How many economy seats does the plane have?

45 economy seats

RP6 Uses intuitive methods to solve ratio and proportion problems (e.g., adding or halving), but cannot generalize these methods to other ratio/proportion problems

example

If you have 12 cups of flour to 3 cups of sugar, how many cups of flour do you have for 5 cups of sugar?

24 cups of flour

Class Profile Instructions

About the Class Profile

Completing an analysis of student work gives you a clear picture of the strategies an individual student is applying to a particular problem or topic in mathematics. Such an analysis is even more powerful when it is applied to the Math Navigator class as a whole.

The Class Profile gives you both. By reading the Class Profile across a row, you can see where each student stands at any point in time. Reading down the columns allows you to see the strengths and needs of the entire class at a glance. By reviewing the Class Profile, you will be able to make decisions that target appropriate instruction to individuals, small groups, and the whole Math Navigator class.

The first pages of the Class Profile provide assessment items related to the content of the module. The last page is based on the mathematical practices from the Common Core State Standards for Mathematics.¹ On this page, record evidence of students using these practices.

Recording Data on the Class Profile

When you see—either through discussion, analysis of student work, or direct observation—that a student understands a concept, still has a misconception, or engages in a mathematical practice, make a note on your Class Profile. As the student's understanding increases, update the Class Profile.

Using the Class Profile

Review the Class Profile periodically during the lesson to help you decide which topics would be most beneficial for your students to focus on during the class discussion. Address topics that most of the students in the Math Navigator group need to learn during the show me, work time, or probing for understanding parts of the lesson. Address topics that only some students are struggling with during partner work or in conferences. If only one or two students need help with a topic, address the topic in an individual conference.

Give a copy of the completed Class Profile to each student's classroom teacher at the end of the module.

¹Common Core State Standards Initiative. 2010. "Common Core State Standards for Mathematics": 6–8. Accessed July 1, 2011. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

CLASS PROFILE (1 OF 3)

Concepts	
1	Student Name
2	C1: Identifies a best solution strategy for a problem based on information given
3	C2: Recognizes and describes patterns in a table
4	C3: Understands speed/time/distance relationship
5	C4: Understands conversion factors to solve problems
6	C5: Draws a diagram based on information in a problem
7	C6: Understands and applies concepts of area, length, or volume
8	C7: Creates original mathematics problems based on given information
9	C8: Analyzes data in a table
10	Observed Errors

Mathematical Practice Standards

- MP1:** Make sense of problems and persevere in solving them.
- MP2:** Reason abstractly and quantitatively.
- MP3:** Construct viable arguments and critique the reasoning of others.
- MP4:** Model with mathematics.
- MP5:** Use appropriate tools strategically.
- MP6:** Attend to precision.
- MP7:** Look for and make use of structure.
- MP8:** Look for and express regularity in repeated reasoning.

Student Name

Observations

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



A Complete Solution to a Word Problem

includes all of the following ...



A written estimate



All work that you do



An equation (even if you solved it using column form)



A diagram, number line, table, or other representation



The answer to the question in a complete sentence



What to Do If You Get Stuck



Look at past work times



Look at the charts that are posted



Model the problem using counters or other materials



Sketch a diagram or other representation



Change the numbers to make the problem simpler



Write what you do know



Write down questions to ask later



Check other resources