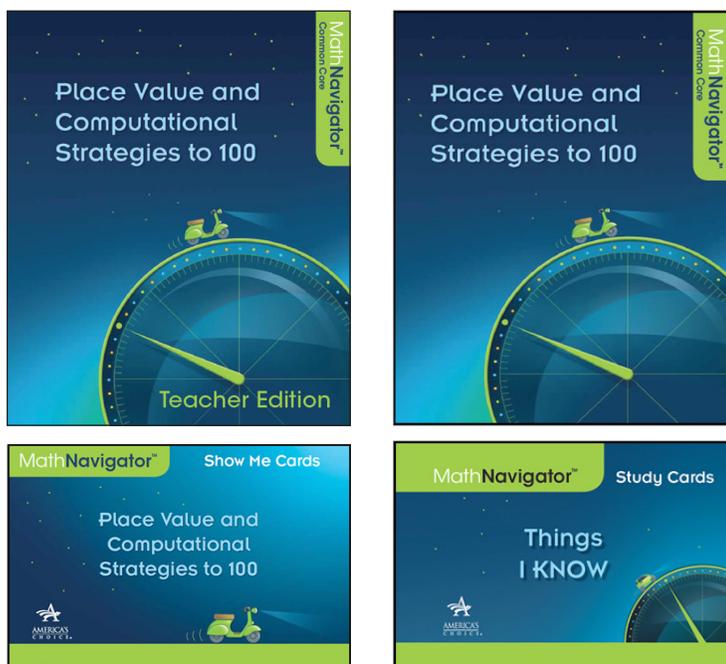


## Research behind the Program

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### Introduction

This guide introduces the research that supports the development of the Math Navigator program and identifies research-based intervention strategies within the program.



### Background

Math Navigator is a research-based program for elementary, middle, and early high school. It is designed to address the needs of struggling students who currently work at grade level, but have difficulty keeping up with the rest of the class. At some point in their academic careers, these students may have received weak preparation in certain concepts, or they may have misunderstood the concepts altogether.

Math Navigator intervention does not repeat initial teaching, nor is it remedial in nature. Instead, the program focuses on revising misconceptions and helping students reconstruct their conceptual framework. Students analyze their work for errors, test the validity of their work against other students' work, try multiple strategies to determine if their prior knowledge holds true or fails them, and modify or invent similar problems. All of this leads to revising misconceptions. In the process, students learn to problem solve, reason mathematically, and justify their work.

**Enlarging and Reducing 6**

**Mathematical Goals**

- Enlarge a shape, maintaining the ratios within the shape, by finding equivalent ratios with larger measures
- Understand the concept of similarity through the idea of enlargement
- See the relationship between scale factors and ratios in general

**Materials Needed**

- Student books and markers
- Response boards
- Study Cards 11 and 12, which correlate with this lesson
- Concept Book pages 255–266, which correlate with this lesson
- Show Me Cards PR 6-1 through PR 6-5
- Class Profile

**Class Profile Assessment**

C3	Understands the concepts of enlarging and reducing while maintaining proportionality
C4	Understands proportionality in the context of similar shapes
P1	Computes and/or compares unit rates
P3	Computes equivalent rates and ratios

**Focus on Language**

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work.

Spanish cognates are in parentheses.

- enlarge (aumentar)
- golden ratio
- similar shapes
- scale factor (factor de escala)
- constant of proportionality

**Misconceptions Addressed**

- Uses additive reasoning for ratios, rates, or proportional relationships
- Does not understand ratio tables

MATH NAVIGATOR 25

**Enlarging and Reducing 6**

**setting the direction**

Have students turn to page 11 in the Student Book. Students will learn to use proportional reasoning to decide which frames would fit a  $5'' \times 7''$  photograph that has been enlarged. The key to understanding this problem is recognizing that the photograph represents a rectangle with sides in proportion to each other in the ratio 5 to 7. An enlargement of the photo preserves this ratio, so Jamal needs to select a frame that has sides with lengths in the same proportion: 5 to 7.

Have students work in pairs to answer the questions and then have a whole-class discussion about the answers. Ask students to name the geometric shape formed by the frame, and tell them that next they will be applying their proportional reasoning skills to a problem in geometry.

**scaffolding for success**

If students think that 5 to 7 is the same as 7 to 9, remind them that equivalent ratios are not calculated by adding a constant factor; they can only be calculated by multiplying by a constant factor. In other words, 5 to 7 is the same as 10 to 14 (multiply by 2), but it is not the same as 7 to 9 (add 2). Use a calculator to show that the ratio  $\frac{5}{7}$  equals  $\frac{10}{14}$  but is not equal to  $\frac{7}{9}$ .

**english language learners**

Make sure students understand that enlarge means to make larger, and an enlargement refers to a photograph that has been made larger. Also point out that reduce means to make smaller and that a photographic reduction is a photograph that has been made smaller.

26 PROPORTIONAL REASONING

## The Six Main Premises

The research that lays the foundation for the development of Math Navigator is derived from Six Main Premises. Read about each of these premises, and discover how experts in math education have influenced the program's philosophy.

### Prior Knowledge and Misconceptions

Math Navigator places an emphasis on prior knowledge and revising misconceptions. Students bring prior knowledge of mathematical concepts to class, and research shows that working with that prior knowledge can lead to a deeper understanding and long-term learning. Malcolm Swan (2005) places great importance on directing students' prior knowledge to problems that have been carefully devised to produce incorrect responses. Wrong answers often reveal misconceptions in ways that students perceive the target concept.

The program uses this strategy because it develops collaborative and active learners whose task is to discover the source of their own misconceptions. Prior knowledge is activated in a way that pulls students out of passive modes of learning, exposes their misconceptions, and requires them to resolve their conflicts through discussion and an active study of the concepts.

**15 Correcting Mistakes**

**show me**  
Write a fraction equivalent to  $\frac{4}{8}$ .

**work time**  
Some students made mistakes when they answered the following problems. For each problem, mark the incorrect answer(s) and explain what mistake the student made.

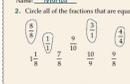
Name: Tran

1. Shade the following figure so that  $\frac{3}{4}$  of the whole rectangle is shaded.



Name: Martha

2. Circle all of the fractions that are equal to 1.



Name: Dwayne

3. Write two fractions that are shown by the point N on this number line.



**15 Correcting Mistakes**

4. Write some advice you would give to one or more of the students to help them avoid making the same mistakes again.

**reflection**

I found (circle one) Tran's    Martha's    Dwayne's    problem easiest to correct because ....

\_\_\_\_\_

\_\_\_\_\_

## Targeted Concepts

Math Navigator taps into vast amounts of research that shows that learners' mistakes in mathematics are often the result of "consistent, alternative interpretations of mathematical ideas" (Swan 2005). For example, most learners generalize from their early experiences that

- you cannot divide smaller numbers by larger ones;
- division always makes numbers smaller;
- the more digits a number has, the larger its value is;
- shapes with bigger areas have bigger perimeters;
- letters represent particular numbers; and
- "equals" means "makes" (Swan 2005).

Interventions can target these misconceptions and their related concepts.

The program engages students in a deeper study of fewer, more critical concepts. It requires students to interact with mathematics, while balancing work with concepts, problem solving, and skills. It accomplishes this by structuring sessions around a handful of tasks that students study deeply. By including only a few problems, Math Navigator provides students with extra time to rework their computations or to try other methods. The inclusion of fewer tasks gives students time to think and form questions.

More Addition and Subtraction Word Problems **3**

**work time**

1. Fill in the missing parts of this table.  
All of the parts should match the problem.  
Check your work with your partner as you go along.



a. **PROBLEM**  
Malaya and Gabby both practiced playing piano last week. Malaya spent 55 more minutes practicing than Gabby did. Gabby practiced 85 minutes. How many minutes did Malaya practice playing the piano last week?

<b>ESTIMATE</b>	<b>UNKNOWN</b> Let ____ =
<b>DIAGRAM, NUMBER LINE, OR OTHER REPRESENTATION</b>	<b>EQUATION</b>
<b>ANSWER</b>	

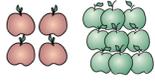
MATH NAVIGATOR 9

## The Algebraic Structure of Arithmetic

In the most successful countries, mathematics instruction emphasizes the algebraic structure of arithmetic even in early elementary instruction. Quite often, early instruction in arithmetic and mathematics fails to take into account the goal of algebra preparedness. According to researcher Liping Ma (1999), even concepts in early arithmetic, such as addition and division, can be presented and developed in ways that lead to easier transitions to algebra. Math Navigator helps students learn mathematics in a way that familiarizes them with the algebraic structure of mathematics. It prepares students for more advanced mathematics, while incorporating the foundational aspects as well.

**10** Adding or Subtracting 9

**work time**

1.  How many apples in all?

2. 17 children were on a bus. Some children got off. Then there were 9 children on the bus. How many children got off?



40 USING ADDITION AND SUBTRACTION TO SOLVE PROBLEMS TO 20

## The Language-Rich Environment

Building technical vocabulary is an important, yet often neglected, part of mathematics instruction. Students should be able to discuss mathematics using the academic language of the discipline and talk that is accountable. Phil Daro (2006) has done extensive research on the benefits of accountable talk in mathematics. Based on Daro's research and philosophy, the program requires students to justify their reasoning and support the accuracy of their knowledge. Daro (2006) argues that discussing the concepts on which procedures are based, as well as the reasons behind the steps in any procedure, can serve to strengthen students' understanding of both.

In Math Navigator, students collaborate and talk about mathematics in a language-rich environment to deepen and accelerate their learning. Then they refine their understanding and make new knowledge gains visible to others.

### Word Problems with Fractions 6

**Mathematical Goals**

- Solve one-step story problems in a variety of measurement contexts
- Understand how to express fractions of measurements
- Relate one measurement to another measurement using fractions

**Class Profile Assessment**

C1	Interprets the meaning of the multiplication and division symbols $\times$ and $\div$
C2	Identifies the operations for solving single-step problems
P1	Connects multiplication and division to diagrams and words
P2	Multiplies and divides simple whole numbers and fractions
P3	Solves simple word problems

**Materials Needed**

- Student Books and markers
- Response board with a marker and eraser (one for each student)
- Calculators (for each student)
- Study Cards 7-8, which correlate with this lesson
- Concept Book pages 149-150 and 152-153, which correlate with this lesson
- Show Me Cards PRN 6-1 through PRN 6-5
- Class Profile

**Optional Materials**

- Large 11"  $\times$  17" sheet of paper recommended (teacher demonstration)

**Misconceptions Addressed**

**ERROR**

- When dividing a fraction by a whole number, only divides the denominator by the whole number
- When dividing a fraction by a fraction, confuses which number is the dividend and which is the divisor
- When multiplying a fraction by a fraction, divides both the numerator and denominator of one fraction by the denominator of the other
- When multiplying a fraction by a fraction, multiplies the denominators, but not the numerators
- Believes that you always divide the larger by the smaller
- Thinks that dividing by one-half is the same as dividing in half

**Focus on Language**

Model the use of these words, and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- gallon (*galón*)
- mile (*milla*)
- minute (*minuto*)
- second (*segundo*)
- inch

### Focus on Language

Model the use of these words, and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- gallon (*galón*)
- mile (*milla*)
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- second (*segundo*)
- inch

## Better Learners of Mathematics

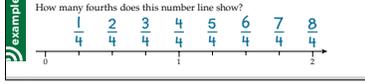
Vygotsky's (1978) research supports the theory that knowledge is not something that individuals gain from the outside; rather it is something that they gain through their own active experiences. This means that learning and knowledge are to a large extent culturally and socially influenced. Vygotsky's research has shown that the ability to learn well goes beyond knowledge or cognitive ability.

It also includes social performances such as asking questions, seeking help, persevering in problem solving, and engaging in mathematical discourse with peers and teachers.

### 14 Whole-Number Fractions

**show me**  
Write a fraction equivalent to  $\frac{3}{6}$ .

**setting the direction**

**example**  
How many fourths does this number line show?  


**work time**

- a. At their work table, 4 students open a box of 4 pencils and share them equally. How many pencils does each student get?

b. Write three different fractions that are all equal to 1. 🎯 If you need help, look back at your work in lesson 11.
- The fraction  $\frac{2}{2}$  is 2 one-halves, and the fraction  $\frac{2}{4}$  is 2 one-fourths.

a. What whole number is  $\frac{2}{1}$ ? 🎯 Use a ruler or number line if you need help.

b. What whole number is  $\frac{4}{2}$ ?

40 FRACTIONS AS NUMBERS

Math Navigator is designed with the notion that intelligence can be learned; as such, the goal of the program is to make students better learners of mathematics. Through the use of rituals and routines and sharing and discussing thinking and strategies with peers and the whole group, the Math Navigator classroom creates an environment that teaches students how to be better learners. Research shows that students learn best when they know what to expect. Routines, rituals, and the premises of each lesson give students confidence to know what teachers expect of them. They also help students become more productive learners who share the attitudes, habits of mind, and learning techniques of effective critical thinkers, which ultimately lead to improved performance.

## Instructional Support

Support for teachers' learning of the instructional design and teaching practices should come from the program materials. The Math Navigator Instructor Edition is designed to provide comprehensive support to teachers by offering detailed instructional guidance, English language learner (ELL) support, and a variety of both formal and informal assessments.

Perhaps the greatest instructional support is the inherent structure of Math Navigator sessions. The rituals, routines, and flow of the sessions do not change from day to day. Once students learn how to engage in solo work, partner work, and whole-group discussions, they are able to do these tasks automatically.

### 3 Fractions of a Piece of Clay

**show me**  
Begin the lesson by using Show Me Cards FAN 3-1 through 3-5. During today's show me, students will compute in their heads the solutions to basic division problems. Have students answer the following questions on their response boards.

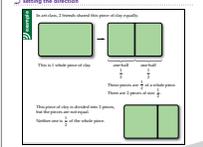
<ul style="list-style-type: none"> <li>Compute in your head ...</li> <li>FAN 3-1 <math>6 \div 3</math> Answer: 2</li> <li>FAN 3-3 <math>8 \div 2</math> Answer: 4</li> <li>FAN 3-5 <math>9 \div 3</math> Answer: 3</li> </ul>	<ul style="list-style-type: none"> <li>FAN 3-2 <math>10 \div 2</math> Answer: 5</li> <li>FAN 3-4 <math>14 \div 2</math> Answer: 7</li> </ul>
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To finish, have students complete the show me problem in the Student Book.

**setting the direction**

**Introducing the Lesson**  
Tell students that today you will continue to examine what it means to share items evenly.  
Remind students that in Lesson 2 they divided a set of eggs or lengths of ribbon into equal-sized groups or lengths. Explain that this lesson is different because they will be dividing an area—a rectangular shape—into equal-sized pieces.  
Read the example aloud to students.

**English Language Learners**  
It is important to explain new terms to students before the terms are used in problems. Remember, ELLs are learning new mathematical concepts and a new language at the same time.

**setting the direction**  


12 FRACTIONS AS NUMBERS

## Common Core State Standards for Mathematics

The Common Core State Standards for Mathematics include Standards for Mathematical Content and Standards for Mathematical Practice. Math Navigator incorporates these standards into students' daily experiences.

## Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that focus on math processes and proficiencies that apply to all areas of mathematics. The Class Profile includes the eight Standards for Mathematical Practice to assist teachers in documenting students' use of these practices. Throughout the lessons, teachers can highlight situations in which students demonstrate use of the Standards for Mathematical Practice in order to encourage all students to engage in the practices.

<b>CLASS PROFILE</b>	<b>Mathematical Practice Standards</b>	<b>MP1:</b> Make sense of problems and persevere in solving them.	<b>MP5:</b> Use appropriate tools strategically.	
		<b>MP2:</b> Reason abstractly and quantitatively.	<b>MP6:</b> Attend to precision.	
		<b>MP3:</b> Construct viable arguments and critique the reasoning of others.	<b>MP7:</b> Look for and make use of structure.	
		<b>MP4:</b> Model with mathematics.	<b>MP8:</b> Look for and express regularity in repeated reasoning.	
	<b>Student Name</b>		<b>Observations</b>	
	1			
	2			
3				
4				
5				
6				

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## Review

This guide introduced the research that supports the development of the Math Navigator program and identified research-based intervention strategies within the program.

## References

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