

# Ratios and Rates



AMERICA'S  
CHOICE.

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**Online Resources**

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## Introduction to Math Navigator

Dear Parent/Guardian,

\_\_\_\_\_ has been selected to participate in Math Navigator! Math Navigator is one of the ways that our school is working to help all students succeed in mathematics. The program gives students the additional time and instruction they need to improve their performance in this important subject.

Your child will be participating in the *Ratios and Rates* module. The main goal of this module is to help students better understand ratios and rates. Students begin the module by contrasting ratio relationships with additive comparisons. They explore the different representations of ratio relationships, including ratio notation, fractions, decimals, percents, words, and tape diagrams. Students learn that ratio relationships can describe part-to-part or part-to-whole situations. Students also find equivalent ratios and write them in their simplest form.

There are a variety of materials students will use with this module: one of them is a set of Study Cards. These cards include mathematical ideas for students to master, game cards, and blank cards that students can customize with concepts that they need to work on. Students are encouraged to use these cards during the lessons, as well as during free time and at home. Please encourage your child to share them with you.

The more enthusiastic you can be about Math Navigator, the more it will help your child. Ask questions each day about what your child learned and how the Math Navigator class was different from your child's regular math class. It is important for you to acknowledge what your child has accomplished both on a day-to-day basis and after completing the Math Navigator module.

We are excited about using Math Navigator with students. Learn more about this special program and how it works by reading the short description that follows. If you have any questions about the program, please do not hesitate to contact us here at school.

### How Math Navigator Works

#### Structure of a Module

Each module contains 20 days of 30- or 45-minute lessons, including a pre-test and post-test. During the 20 days, students have two or three checkpoint lessons that assess their understanding of the concepts in the module.

#### Frequent Skills Practice

Most lessons include a Show Me session in which students practice and reinforce skills. It is also a time for students to learn strategies and techniques that make computation easier.

#### Emphasis on Understanding

The lessons are carefully designed to uncover mistakes that result from students misunderstanding something. We call such mistakes *misconceptions*. Misconceptions need to be corrected because they can interfere with new learning. Math Navigator modules do not attempt to reteach everything that students have learned about a topic. Instead, they help students understand the mathematics of the procedures and concepts that they have already learned so that they can correct the misconceptions that are getting in the way of their progress.

#### Learning to Think Mathematically

Lessons are structured to teach students to think like mathematicians. Students will learn how to ask themselves questions before beginning a problem; to use diagrams, tables, and other methods of representing problems; and to estimate as a way of determining whether their answers are reasonable. Most importantly, they will come to see that mistakes are opportunities for learning, rather than something to hide.

## Misconceptions and Errors

<b>A1</b>	Confuses the two axes (or two coordinates) of a graph
<b>A24</b>	Compares graph features, such as steepness, even though they do not use the same units on both axes
<b>A25</b>	Chooses inappropriate units for graph axes
<b>D14</b>	Multiplies or divides correctly but incorrectly places the decimal point
<b>E9</b>	Misapplies the properties of equality
<b>F1</b>	Writes a fraction or a probability as part-to-part instead of part-to-whole
<b>F4</b>	Writes a fraction or a probability as whole-to-part not part-to-whole
<b>F21</b>	Does not understand the concept of equivalence
<b>F23</b>	Orders fractions based on the numerators only
<b>F24</b>	Orders fractions based on the denominators only
<b>M13</b>	Uses wrong data
<b>M18</b>	Does not understand or ignores unit conversion
<b>NL9</b>	Incorrectly reads tick marks on a number line
<b>O4</b>	Does not recognize a subtraction situation
<b>O8</b>	Multiplies or divides incorrectly or misapplies appropriate procedures for multiplying or dividing
<b>O29</b>	Does not understand ratios
<b>O39</b>	Does not estimate to verify answer is reasonable
<b>RP1</b>	Uses additive reasoning for ratios, rates, or ratio relationships
<b>RP2</b>	Does not understand ratio tables
<b>RP3</b>	Does not understand the definition of <i>rate</i>

**A1 Confuses the two axes (or two coordinates) of a graph**

**example**

Which point on the graph represents heavy rainfall and cold temperature?

Point A

**A24 Compares graph features, such as steepness, even though they do not use the same units on both axes**

**example**

Which graph has the greater rate?

$v = 4t$

$d = 35h$

The graph of volume-time shows a larger rate because it has a slope of 4, while the distance-time graph has a slope of 35.

**A25 Chooses inappropriate units for graph axes**

**example** Patti ran 2.5 kilometers in 30 minutes. Sketch a graph showing her distance over time.

**D14 Multiplies or divides correctly but incorrectly places the decimal point**

**example** Lisa's car goes 300 miles on 15 gallons of gas. How fuel efficient is Lisa's car?

$$\frac{15 \text{ gallons}}{300 \text{ miles}} = \frac{0.5 \text{ gallons}}{\text{miles}}$$

**E9 Misapplies the properties of equality**

Student confuses negative signs when adding and subtracting terms

**example**

$$2x + 12 = x$$

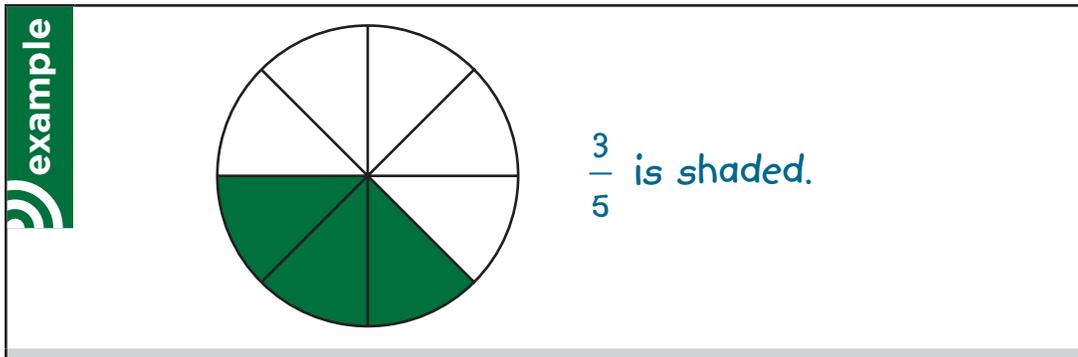
$$x = 12$$

$$2x + 3 = x + 4$$

$$x = 7$$

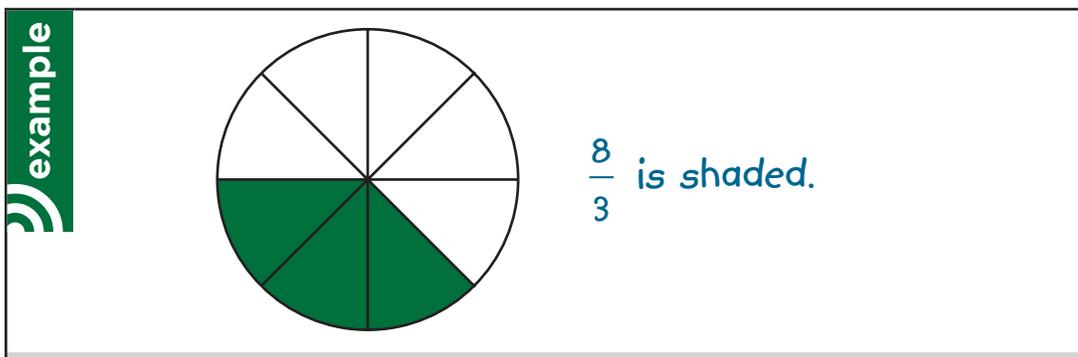
## F1 Writes a fraction or a probability as part-to-part instead of part-to-whole

Students count the diagrams/number of parts and puts the smaller number of parts (or the shaded part) over the larger number of parts, instead of over the whole.



## F4 Writes a fraction or a probability as whole-to-part not part-to-whole

Students count the number of parts of the whole and uses the whole as the numerator.



**F21 Does not understand the concept of equivalence**

example

To make 4 pancakes, it takes  $\frac{1}{2}$  cup of flour and  $\frac{1}{4}$  cup of butter.  
 How much flour and butter is needed for 12 pancakes?  
 Review Samuel's work.

4 pancakes + 4 pancakes + 4 pancakes = 12 pancakes

$\frac{1}{2}$  cup +  $\frac{1}{2}$  cup +  $\frac{1}{2}$  cup = 1  $\frac{1}{2}$  cups of flour

$\frac{1}{4}$  cup +  $\frac{1}{4}$  cup +  $\frac{1}{4}$  cup =  $\frac{3}{4}$  cups of butter

Samuel is wrong because ratios require multiplicative reasoning, and he used addition.

**F23 Orders fractions based on the numerators only**

Students overgeneralize knowledge of whole numbers and ignore the denominator.

example

$\frac{3}{8} > \frac{1}{2}$  because 3 is bigger than 1

**F24 Orders fractions based on the denominator only**

Students overgeneralize knowledge of whole numbers and think that larger denominators reflect larger fractions. Students overgeneralize the idea that “the bigger the denominator, the smaller the part” by ignoring numerators when comparing fractions.

example

Which is greater,  $\frac{1}{8}$  or  $\frac{1}{6}$ ? Explain why.

$\frac{1}{8} > \frac{1}{6}$  because 8 is bigger than 6

$\frac{1}{4} > \frac{3}{5}$  because fourths are greater than fifths

**M13 Uses the wrong units**

Students use the wrong notation or labels or choose the inappropriate unit of measure or the inappropriate measuring tool for the task.

example

Write a unit that would be a good estimate for the weight of an apple.

feet

**M18 Does not understand or ignores unit conversion**

example

Katie spent 2 hours cleaning her room. Then she spent 75 minutes watching a video. How much time did Katie spend cleaning her room and watching a video?

Katie spent 2.75 hours watching the video and cleaning her room.



**O29 Does not understand ratios**

Students may be uncertain about how to write ratios, especially when those ratios involve fractions or more than two quantities.

**example**

There are 3 apples, 3 oranges, and 4 potatoes.  
Write the ratio using ratio notation.

$\frac{3}{4}$ , 3 to 4, 3:4

**O39 Does not estimate to verify the answer is reasonable**

**example**

At the store, 4 pounds of broccoli costs \$5. Alan has to buy 5 pounds of broccoli. How much will that cost?

$\frac{4}{5} \times 5 = \$4$

**RP1 Uses additive reasoning for ratios, rates, or proportional relationships**

**example**

There are 25 first class seats and 50 economy class seats on an airplane. The airline has a smaller plane with 20 first class and the same ratio of first class to economy seats. How many economy seats does the smaller plane have?

45 seats

**RP2 Does not understand ratio tables**

**example** Fill in the missing cells of the ratio table.

2	4	6	12
8	10	12	16

**RP3 Does not understand the definition of *rate***

**example** Does this statement describe a rate?

The time it takes a person to walk a kilometer

**No, it is not a rate.**

## Class Profile Instructions

### About the Class Profile

Completing an analysis of student work gives you a clear picture of the strategies an individual student is applying to a particular problem or topic in mathematics. Such an analysis is even more powerful when it is applied to the Math Navigator class as a whole.

The Class Profile gives you both. By reading the Class Profile across a row, you can see where each student stands at any point in time. Reading down the columns allows you to see the strengths and needs of the entire class at a glance. By reviewing the Class Profile, you will be able to make decisions that target appropriate instruction to individuals, small groups, and the whole Math Navigator class.

The first pages of the Class Profile provide assessment items related to the content of the module. The last page is based on the mathematical practices from the Common Core State Standards for Mathematics.<sup>1</sup> On this page, record evidence of students using these practices.

### Recording Data on the Class Profile

When you see—either through discussion, analysis of student work, or direct observation—that a student understands a concept, still has a misconception, or engages in a mathematical practice, make a note on your Class Profile. As the student’s understanding increases, update the Class Profile.

### Using the Class Profile

Review the Class Profile periodically during the lesson to help you decide which topics would be most beneficial for your students to focus on during the class discussion. Address topics that most of the students in the Math Navigator group need to learn during the show me, work time, or probing for understanding parts of the lesson. Address topics that only some students are struggling with during partner work or in conferences. If only one or two students need help with a topic, address the topic in an individual conference.

Give a copy of the completed Class Profile to each student’s classroom teacher at the end of the module.

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<sup>1</sup>Common Core State Standards Initiative. 2010. “Common Core State Standards for Mathematics”: 6–8. Accessed July 1, 2011. [http://www.corestandards.org/assets/CCSSI\\_Math%20Standards.pdf](http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf).





# CLASS PROFILE (3 OF 3)

## Mathematical Practice Standards

- MP1:** Make sense of problems and persevere in solving them.
- MP2:** Reason abstractly and quantitatively.
- MP3:** Construct viable arguments and critique the reasoning of others.
- MP4:** Model with mathematics.
- MP5:** Use appropriate tools strategically.
- MP6:** Attend to precision.
- MP7:** Look for and make use of structure.
- MP8:** Look for and express regularity in repeated reasoning.

**Student Name**

**Observations**

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



# A Complete Solution to a Word Problem

includes all of the following ...



A written estimate



All work that you do



An equation (even if you solved it using column form)



A diagram, number line, table, or other representation



The answer to the question in a complete sentence



# What to Do If You Get Stuck



Look at past work times



Look at the charts that are posted



Model the problem using counters or other materials



Sketch a diagram or other representation



Change the numbers to make the problem simpler



Write what you do know



Write down questions to ask later



Check other resources