

# Positive and Negative Numbers



AMERICA'S  
CHOICE.

**SAVVAS**  
LEARNING COMPANY

Boston, Massachusetts

Chandler, Arizona

Glenview, Illinois

Upper Saddle River, New Jersey

**Online Resources**

This work is protected by United States copyright laws and is provided *solely for the use of teachers and administrators* in teaching courses and assessing student learning in their classes and schools. Dissemination or sale of *any* part of this work (including on the World Wide Web) will destroy the integrity of the work and is *not* permitted.

---

**Copyright © 2012 Savvas Education, Inc., or its affiliate(s). All Rights Reserved.** Printed in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. The publisher hereby grants permission to reproduce these pages, in part or in whole, for classroom use only, the number not to exceed the number of students in each class. Notice of copyright must appear on all copies. For information regarding permissions, write to Savvas Curriculum Group Rights & Permissions, One Lake Street, Upper Saddle River, New Jersey 07458.

America's Choice, the America's Choice A logo, Math Navigator, the Savvas logo are trademarks, in the U.S. and/or other countries, of Savvas Learning Co. or its affiliate(s).

Lesson 1	Letter to Parents	1
Lesson 2	The Number Line	3
Lesson 17	Number Tree	4
	Misconceptions	5
	Class Profile	14
	A Complete Solution to a Word Problem	18
	What to Do If You Get Stuck	19

## Introduction to Math Navigator

Dear Parent/Guardian,

\_\_\_\_\_ has been selected to participate in Math Navigator! Math Navigator is one of the ways that our school is working to help all students succeed in mathematics. The program gives students the additional time and instruction they need to improve their performance in this important subject.

Your child will be participating in the *Positive and Negative Numbers* module. The main goal of this module is to help students understand that negative numbers are a reasonable and usable extension of the set of positive numbers and to give students experience in operating with positive and negative numbers. The lessons are designed to lead students through a series of tasks that draw out their misconceptions and errors. Some misconceptions that are explored include believing that negative numbers get “bigger” as you go down the number line and that students must focus on “operation signs” (add and subtract) in calculations but can ignore “number signs” (positive and negative).

There are a variety of materials students will use with this module: one of them is a set of Study Cards. These cards include mathematical ideas for students to master, and blank cards that students can customize with concepts that they need to work on. Students are encouraged to use these cards during the lessons, as well as during free time and at home. Please encourage your child to share them with you.

The more enthusiastic you can be about Math Navigator, the more it will help your child. Ask questions each day about what your child learned and how the Math Navigator class was different from your child’s regular math class. It is important for you to acknowledge what your child has accomplished both on a day-to-day basis and after completing the Math Navigator module.

We are excited about using Math Navigator with students. Learn more about this special program and how it works by reading the short description that follows. If you have any questions about the program, please do not hesitate to contact us here at school.

### How Math Navigator Works

#### Structure of a Module

Each module contains 20 days of 30- or 45-minute lessons, including a pre-test and post-test. During the 20 days, students have two or three checkpoint lessons that assess their understanding of the concepts in the module.

#### Frequent Skills Practice

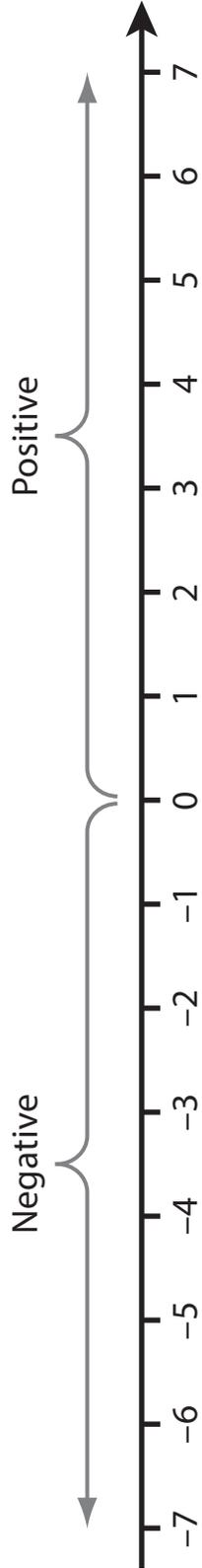
Most lessons include a Show Me session in which students practice and reinforce skills. It is also a time for students to learn strategies and techniques that make computation easier.

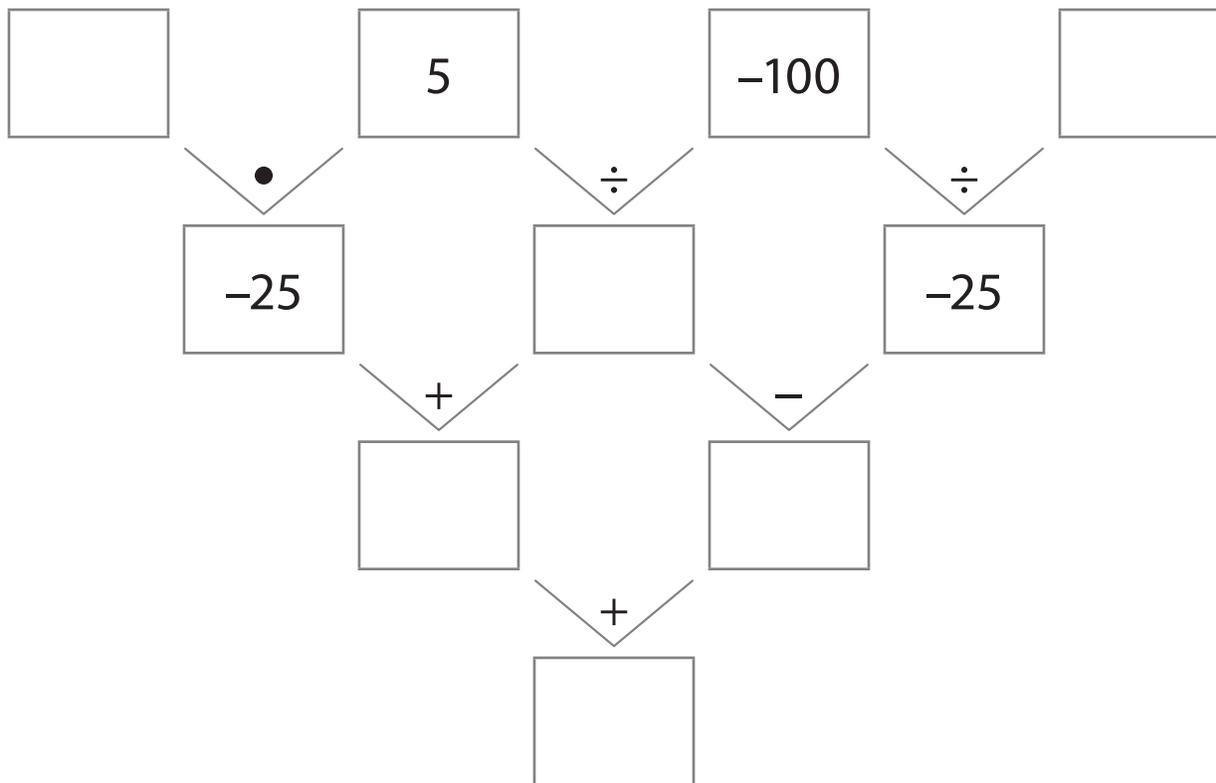
#### Emphasis on Understanding

The lessons are carefully designed to uncover mistakes that result from students misunderstanding something. We call such mistakes *misconceptions*. Misconceptions need to be corrected because they can interfere with new learning. Math Navigator modules do not attempt to reteach everything that students have learned about a topic. Instead, they help students understand the mathematics of the procedures and concepts that they have already learned so that they can correct the misconceptions that are getting in the way of their progress.

#### Learning to Think Mathematically

Lessons are structured to teach students to think like mathematicians. Students will learn how to ask themselves questions before beginning a problem; to use diagrams, tables, and other methods of representing problems; and to estimate as a way of determining whether their answers are reasonable. Most importantly, they will come to see that mistakes are opportunities for learning, rather than something to hide.





## Misconceptions and Errors

Note: Some of these misconceptions are difficult to determine without a conversation with the student. For instance, if students answer  $(-7) + (-3) = 10$ , you don't know if they are taking the absolute values of the numbers or using the idea that two negatives make a positive. You would need to listen to each student's thinking to determine the exact misconception. In assessments, NN2 is used for those situations.

<b>NN1</b>	Does not apply the correct sign to the solution when dealing with operations with negative numbers
<b>NN2</b>	Uses the absolute values of numbers, ignoring their signs
<b>NN3</b>	Does not understand how to order integers or real numbers
<b>NN4</b>	Misapplies sign rules when multiplying or dividing
<b>NN5</b>	Does not understand opposite
<b>NN6</b>	Does not use the correct sign for at least one of the numbers when adding or subtracting
<b>NN7</b>	Does not correctly relate adding negatives with subtracting, or subtracts a negative as a positive
<b>NN8</b>	Does not understand how negative numbers affect inequalities
<b>NN9</b>	Does not understand that $-a = (-1) \cdot a$
<b>NN10</b>	Thinks that properties of operations apply only to positive numbers
<b>NN11</b>	Thinks that negative numbers are not the same as "ordinary" numbers because you cannot use them for counting
<b>NN12</b>	When comparing numbers, thinks that the number with more digits is greater
<b>NN13</b>	Does not recognize a multiplication situation
<b>NN14</b>	Misapplies the multiplication rule "two negatives make a positive" when adding or subtracting
<b>NN15</b>	Misapplies the multiplication rule "negative by positive makes negative" when adding or subtracting
<b>NN16</b>	Subtracts if the second number is negative but ignores the sign of the first number
<b>NN17</b>	Does not understand addition and subtraction on a number line
<b>E1</b>	Fails to apply or misapplies order of operations principles
<b>E8</b>	Does not understand the equals sign; for example, reads the equals sign as "makes" without considering what is on the other side of the equation
<b>EXP8</b>	Confuses negative numbers with negative roots
<b>NL11</b>	Does not understand number lines or how to count on number lines
<b>O1</b>	Does not use the correct operation
<b>O24</b>	Does not recognize or misapplies the distributive property of multiplication

**NN1 Does not apply the correct sign to the solution when dealing with operations with negative numbers**

**example**

$$-7 + 3 = 4$$

$$-7 \cdot -3 = -21$$

**NN2 Uses the absolute values of numbers, ignoring their signs**

**example**

$$-7 + -3 = 10$$

$$-8 \div 2 = 4$$

$$(-11) - 6 = 5$$

$$7 - (-2) = 5$$

**NN3 Does not understand how to order integers or real numbers**

**example**

Which of these numbers are greater than 2?

0   -3   5   -12   12   -2.5

**0, 5, and 12**

Arrange these numbers from least to greatest

-9   9   -0.9   0.9

**-0.9, -9, 0.9, 9**

**NN4 Misapplies sign rules when multiplying or dividing**

**example**

$$-7 \cdot -3 = -21$$

$$-8 \div -2 = -4$$
**NN5 Does not understand opposite**

**example**

Which of whole numbers does not have its opposite in this set?

$$\{4, 102, -4, 5, -10, 10, -102, \frac{1}{5}, \frac{1}{10}\}$$

4

**NN6 Does not use the correct sign for at least one of the numbers when adding or subtracting**

**example**

$$(-4) + (-6) = 2$$

$$14 - 12 - 14 - 12 = -4$$
**NN7 Does not correctly relate adding negatives with subtracting or subtracting a negative as a positive**

**example**

Write a subtraction that is equivalent to the addition  $-13 + (-19)$ .

$$13 - 19$$

$$(-20) + (-18) - (-6) = 4$$

**NN8 Does not understand how negative numbers affect inequalities**

**example**

Solve:  $4 - 3g > -11$   
 $- 3g > -15$   
 $g > 5$

**NN9 Does not understand that  $-a = (-1) \cdot a$** 

**example**

Simplify:  $-(9 - x)$   
 $9 - x$

**NN10 Thinks that properties of operations apply only to positive numbers**

**example**

Solve:  $3 - 5$   
 $3 - 5 = 5 - 3 = 2$

**NN11 Thinks that negative numbers are not the same as “ordinary” numbers because you cannot use them for counting**

example

Which of these statements do you agree with?

Jamal said: “I think negative numbers are like ordinary numbers but with a negative sign.”

Dwayne said: “Negative numbers are like debts, when you owe money, and positive numbers are like credits, when you have money.”

Tom said, “I do not think there is any such thing as a negative number. You can’t count negative numbers.”

*I agree with Tom and Dwayne. You can have debts but you don’t need to pay them off, so there really is not a negative number.*

**NN12 When comparing numbers, thinks that the number with more digits is greater**

example

Which is greater,  $-213$  or  $-21$ ?

$-213 > -21$

**NN13 Does not recognize a multiplication situation**

example

Misha bought 11.7 gallons of diesel fuel at \$2.34 per gallon. How much did he pay for the fuel?

*Misha paid \$14.04.*

**NN14 Misapplies the multiplication rule “two negatives make a positive” when adding or subtracting**

**example**

$$(-7) + (-4) = 11$$

$$(-6) - 4 = 10$$

**NN15 Misapplies the multiplication rule “negative by positive makes negative” when adding or subtracting**

**example**

$$(-7) + (4) = -11$$

$$25 - (-4) = -29$$

**NN16 Subtracts if the second number is negative but ignores the sign of the first number**

**example**

$$(-7) + (-4) = 3$$

$$(-10) - (-3) = 7$$

$$(-6) - (-4) = 2$$

**NN17 Does not understand addition and subtraction on a number line**

**example**

Sketch a number line that shows the addition  $(-3) + (-1)$ ?

The diagram shows a horizontal number line with tick marks from -5 to 5. A blue arc is drawn above the line, starting at the point -3 and ending at the point -1. A blue arrow labeled 'sum' is drawn above the line, starting at the point -1 and pointing to the right towards the point 0.

## E1 Fails to apply or misapplies order of operations principles

**example**

$$(-40) - [(-8) \div 4] = 8$$

$$[(-8) \cdot (-2)] + (-2) = 32$$

## E8 Does not understand the equals sign; for example, reads the equals sign as “makes” without considering what is on the other side of the equation

**example**

Choose the correct symbol to fill in the blank.

$$8 \text{ \_\_\_ } -8$$

$$8 = -8$$

## EXP8 Confuses negative numbers with negative roots

**example**

Simplify.

$$2 - (3^2)$$

$$2 - (3^2) = 2 + (-3)^2 = 2 + 9 = 11$$

**NL11 Does not understand number lines or how to count on number lines**

**example**

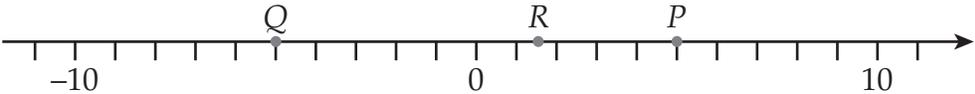
Which statement is true?

- A. 4 is less than  $-8$ .
- B. On the number line,  $-4$  is twice as far from 0 as 8 is from 0.
- C. On the number line, 4 is farther from 0 than  $-8$  is from 0.
- D. 8 is less than  $-4$ .

**C. On the number line, 4 is farther from 0 than  $-8$  is from 0.**

**example**

Which letter(s) represents a negative number on this number line?



**Point R is negative.**

**O1 Does not use the correct operation**

**example**

What is the unknown number in this equation?

6.5  $\div (-6) = 0.5$

**O24 Does not recognize or misapplies the distributive property of multiplication**

**example** Write an expression equivalent to  $-8(4 - 2x)$   
 $(-32) - 2x$

**example** Given values for  $a$ ,  $b$  and  $c$ , write  $a(b + c)$  and  $a \cdot b + a \cdot c$

$a$	$b$	$c$	$a \cdot (b+c)$	$a \cdot b + a \cdot c$
-4	-7	-1	$-4 \cdot (-7 + -1) = 32$	$-4 \cdot -7 + -4 \cdot -1 = 32$
3	-2	-5	$3 \cdot (-2 + -5) = -21$	$3 \cdot -2 + 3 \cdot -5 = -21$
-1.75	2.5	-3.5	$-1.75(2.5 + -3.5) = 1.75$	$-1.75 \cdot 2.5 + -1.75 \cdot -3.5$
-15	2	4	$-15 \times (2 + 4) = -90$	$-15 \cdot 2 + -15 \cdot 4 = -90$

b. yes,  $a \cdot b + a \cdot c$ .

## Class Profile Instructions

### About the Class Profile

Completing an analysis of student work gives you a clear picture of the strategies an individual student is applying to a particular problem or topic in mathematics. Such an analysis is even more powerful when it is applied to the Math Navigator class as a whole.

The Class Profile gives you both. By reading the Class Profile across a row, you can see where each student stands at any point in time. Reading down the columns allows you to see the strengths and needs of the entire class at a glance. By reviewing the Class Profile, you will be able to make decisions that target appropriate instruction to individuals, small groups, and the whole Math Navigator class.

The first pages of the Class Profile provide assessment items related to the content of the module. The last page is based on the mathematical practices from the Common Core State Standards for Mathematics.<sup>1</sup> On this page, record evidence of students using these practices.

### Recording Data on the Class Profile

When you see—either through discussion, analysis of student work, or direct observation—that a student understands a concept, still has a misconception, or engages in a mathematical practice, make a note on your Class Profile. As the student's understanding increases, update the Class Profile.

### Using the Class Profile

Review the Class Profile periodically during the lesson to help you decide which topics would be most beneficial for your students to focus on during the class discussion. Address topics that most of the students in the Math Navigator group need to learn during the show me, work time, or probing for understanding parts of the lesson. Address topics that only some students are struggling with during partner work or in conferences. If only one or two students need help with a topic, address the topic in an individual conference.

Give a copy of the completed Class Profile to each student's classroom teacher at the end of the module.

---

<sup>1</sup>Common Core State Standards Initiative. 2010. "Common Core State Standards for Mathematics": 6–8. Accessed July 1, 2011. [http://www.corestandards.org/assets/CCSSI\\_Math%20Standards.pdf](http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf).

# CLASS PROFILE (1 OF 3)

Student Name	C1: Understands the concept of negative numbers	C2: Recognizes that numbers decrease in value as you go left on the horizontal number line and down on the vertical number line	C3: Recognizes opposite signs of numbers as indicating locations on opposite sides of 0 on the number line, and that the opposite of the opposite of a number is the number itself	C4: Interprets the order of positive and negative numbers as their positions on the number line	C5: Understands absolute value of a positive or negative number as distance from 0 on a number line or as magnitude for a positive or negative quantity in a real-world situation	C6: Represents adding and subtracting positive and negative numbers on a number line	C7: Understands that subtracting a positive or negative number is the same as adding its opposite	C8: Demonstrates understanding of why a positive times a negative is negative, and why a negative times a negative is positive	C9: Applies properties of operations when adding and subtracting positive and negative numbers	C10: Uses negative numbers in real-world situations	Observed Errors
1											
2											
3											
4											
5											
6											
7											
8											
9											

# CLASS PROFILE (2 OF 3)

Student Name	1	2	3	4	5	6	7	8	9
<b>P1:</b> Finds and positions positive and negative numbers on a number line									
<b>P2:</b> Compares and orders positive and negative numbers									
<b>P3:</b> Uses inequality symbols to compare and order positive and negative numbers									
<b>P4:</b> Adds positive and negative numbers									
<b>P5:</b> Subtracts positive and negative numbers									
<b>P6:</b> Multiplies and divides positive and negative numbers									
<b>P7:</b> Simplifies expressions involving positive and negative numbers									
<b>Observed Errors</b>									

<b>Mathematical Practice Standards</b>	
<p><b>MP1:</b> Make sense of problems and persevere in solving them.</p> <p><b>MP2:</b> Reason abstractly and quantitatively.</p> <p><b>MP3:</b> Construct viable arguments and critique the reasoning of others.</p> <p><b>MP4:</b> Model with mathematics.</p>	<p><b>MP5:</b> Use appropriate tools strategically.</p> <p><b>MP6:</b> Attend to precision.</p> <p><b>MP7:</b> Look for and make use of structure.</p> <p><b>MP8:</b> Look for and express regularity in repeated reasoning.</p>
<b>Student Name</b>	<b>Observations</b>
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



# A Complete Solution to a Word Problem

includes all of the following ...



A written estimate



All work that you do



An equation (even if you solved it using column form)



A diagram, number line, table, or other representation



The answer to the question in a complete sentence



# What to Do If You Get Stuck



Look at past work times



Look at the charts that are posted



Model the problem using counters or other materials



Sketch a diagram or other representation



Change the numbers to make the problem simpler



Write what you do know



Write down questions to ask later



Check other resources