

# Positive Rational Numbers



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Online Resources

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## Introduction to Math Navigator

Dear Parent/Guardian,

\_\_\_\_\_ has been selected to participate in Math Navigator! Math Navigator is one of the ways that our school is working to help all students succeed in mathematics. The program gives students the additional time and instruction they need to improve their performance in this important subject.

Your child will be participating in the *Positive Rational Numbers* module. The main goal of this module is to help students better understand computations with positive rational numbers. Students will perform computations, particularly multiplication and division, with whole numbers and fractions. Students will interpret situations in which it makes sense to multiply by a fraction and in which it makes sense to divide by a fraction. They will match computations to word problems. In doing so, they will better understand the operations of multiplication and division and better understand what these operations mean when fractions are involved.

There are a variety of materials students will use with this module: one of them is a set of Study Cards. These cards include mathematical ideas for students to master, game cards, and blank cards that students can customize with concepts that they need to work on. Students are encouraged to use these cards during the lessons, as well as during free time and at home. Please encourage your child to share them with you.

The more enthusiastic you can be about Math Navigator, the more it will help your child. Ask questions each day about what your child learned and how the Math Navigator class was different from your child's regular math class. It is important for you to acknowledge what your child has accomplished both on a day-to-day basis and after completing the Math Navigator module.

We are excited about using Math Navigator with students. Learn more about this special program and how it works by reading the short description that follows. If you have any questions about the program, please do not hesitate to contact us here at school.

### How Math Navigator Works

#### Structure of a Module

Each module contains 20 days of 30- or 45-minute lessons, including a pre-test and post-test. During the 20 days, students have two or three checkpoint lessons that assess their understanding of the concepts in the module.

#### Frequent Skills Practice

Most lessons include a Show Me session in which students practice and reinforce skills. It is also a time for students to learn strategies and techniques that make computation easier.

#### Emphasis on Understanding

The lessons are carefully designed to uncover mistakes that result from students misunderstanding something. We call such mistakes *misconceptions*. Misconceptions need to be corrected because they can interfere with new learning. Math Navigator modules do not attempt to reteach everything that students have learned about a topic. Instead, they help students understand the mathematics of the procedures and concepts that they have already learned so that they can correct the misconceptions that are getting in the way of their progress.

#### Learning to Think Mathematically

Lessons are structured to teach students to think like mathematicians. Students will learn how to ask themselves questions before beginning a problem; to use diagrams, tables, and other methods of representing problems; and to estimate as a way of determining whether their answers are reasonable. Most importantly, they will come to see that mistakes are opportunities for learning, rather than something to hide.

## Misconceptions and Errors

<b>D28</b>	Counts the digits to the right of the decimal point
<b>F27</b>	When dividing fractions, divides the second number by the first or inverts both numbers
<b>F40</b>	When multiplying fractions, multiplies the numerator of the first fraction by the denominator of the second, and adds the product of the denominator of the first and the numerator of the second
<b>F41</b>	When dividing fractions, multiplies instead
<b>F42</b>	When dividing fractions, divides the numerators and divides the denominators
<b>F46</b>	Thinks that dividing by one-half is the same as dividing in half
<b>M5</b>	Does not know formulas for volume or surface area
<b>O5</b>	Does not recognize a multiplication situation
<b>O6</b>	Does not recognize a division situation
<b>O7</b>	Thinks that when dividing, you always divide by the smaller number
<b>O8</b>	Multiplies or divides incorrectly or misapplies appropriate procedures for multiplying or dividing
<b>O10</b>	Confuses the parts of a division problem such as dividend and divisor
<b>O12</b>	Thinks that multiplying always makes things bigger
<b>O13</b>	Thinks that dividing always makes things smaller
<b>O14</b>	Does not recognize or misapplies the commutative property
<b>O52</b>	Confuses greatest common factor and least common multiple

**D28 Counts the digits to the right of the decimal point**

Counting digits instead of estimating demonstrates a “blind reliance on rules.”

**example**

Identify if the calculation  $12.4 \times 4.5 = 55.8$  is incorrect by estimating the value you expect for the answer. Explain why it is incorrect

*It is incorrect because the correct answer will have two digits to the right of the decimal point.*

**F27 When dividing fractions, divides the second number by the first or inverts both numbers**

**example**

Solve.

$$\frac{1}{2} \div \frac{1}{6} = 12$$

**F40 When multiplying fractions, multiplies the numerator of the first fraction by the denominator of the second, and adds the product of the denominator of the first and the numerator of the second**

**example**

$$\frac{3}{4} \times \frac{4}{5} = 3 \cdot 5 + 4 \cdot 4 = 31$$

**F41 When dividing fractions, multiplies instead**

**example**

$$\frac{3}{4} \div \frac{4}{5} = \frac{12}{20}$$

**F42** When dividing fractions, divides the numerators and divides the denominators

**example**

$$\frac{3}{4} \div \frac{6}{24} = \frac{2}{6} \text{ or } \frac{1}{3}$$

**F46** Thinks that dividing by one-half is the same as dividing in half

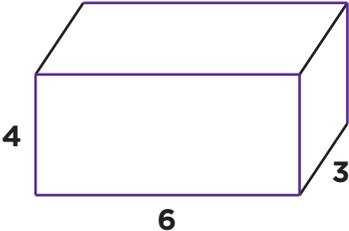
**example**

15 divided by one-half is  $\frac{15}{2} = 7.5$

**M5** Does not know formulas for volume or surface area

**example**

Find the volume of this box:



$6 \times 3 + 6 + 6 + 4 + 4$

**O5** Does not recognize a multiplication situation

**example**

Which expression shows the cost of renting a bicycle at \$1.50 per hour for  $x$  hours?

$x + 1.5$  dollars

**O6 Does not recognize a division situation**example

Write a situation that corresponds to this equation.  $y = \frac{12}{x}$

*Angela is twelve years older than Carlo. Angela is  $x$  years old; Carlo is  $y$  years old.*

**O7 Thinks that when dividing, you always divide by the smaller number**

The student bases all his answers on whole numbers and divides by the smaller number. He does not realize that fractions represent a division.

example

Mr. Hakim wants to share 4 sandwiches equally among 8 students. How many sandwiches does each student get?

*Each student gets 2 sandwiches.*

**O8 Multiplies or divides incorrectly or misapplies appropriate procedures for multiplying or dividing**

The student divides the whole expression by the denominator rather than just the part that is the fraction, or does not divide the whole expression

**example**

$196 \div 14 \cdot 3 = 39$

What number makes this equation true?

$10 \div 5 = 25 \div 10$

**example**

Equation 3

Finish

$\frac{x+7}{5} = 25$

- 2

$\frac{x+9}{5} = 27$

· 3

$\frac{x}{5} + 3 = 9$

+ 3

$\frac{x}{5} = 6$

÷ 5

Start

$x = 30$

**O12 Thinks that multiplying always makes things bigger**

**example**

Estimate the answer to  $30 \times \frac{16}{17}$ .

Greater than 30

**O13 Thinks that dividing always makes things smaller**

**example** Estimate the answer to  $25 \div \frac{3}{4}$ .  
Less than 25

**O14 Does not recognize or misapplies the commutative property**

Student thinks subtraction is commutative

**example**  $5 - 3 = 3 - 5$

Student thinks division is commutative

**example**  $5 \div 3 = 3 \div 5$

**O52 Confuses greatest common factor and least common multiple**

**example** Find the greatest common factor of 6 and 9.  
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## Class Profile Instructions

### About the Class Profile

Completing an analysis of student work gives you a clear picture of the strategies an individual student is applying to a particular problem or topic in mathematics. Such an analysis is even more powerful when it is applied to the Math Navigator class as a whole.

The Class Profile gives you both. By reading the Class Profile across a row, you can see where each student stands at any point in time. Reading down the columns allows you to see the strengths and needs of the entire class at a glance. By reviewing the Class Profile, you will be able to make decisions that target appropriate instruction to individuals, small groups, and the whole Math Navigator class.

The first pages of the Class Profile provide assessment items related to the content of the module. The last page is based on the mathematical practices from the Common Core State Standards for Mathematics.<sup>1</sup> On this page, record evidence of students using these practices.

### Recording Data on the Class Profile

When you see—either through discussion, analysis of student work, or direct observation—that a student understands a concept, still has a misconception, or engages in a mathematical practice, make a note on your Class Profile. As the student’s understanding increases, update the Class Profile.

### Using the Class Profile

Review the Class Profile periodically during the lesson to help you decide which topics would be most beneficial for your students to focus on during the class discussion. Address topics that most of the students in the Math Navigator group need to learn during the show me, work time, or probing for understanding parts of the lesson. Address topics that only some students are struggling with during partner work or in conferences. If only one or two students need help with a topic, address the topic in an individual conference.

Give a copy of the completed Class Profile to each student’s classroom teacher at the end of the module.

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<sup>1</sup>Common Core State Standards Initiative. 2010. “Common Core State Standards for Mathematics”: 6–8. Accessed July 1, 2011. [http://www.corestandards.org/assets/CCSSI\\_Math%20Standards.pdf](http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf).

# CLASS PROFILE (1 OF 3)

Concepts	
Student Name	1
	2
	3
	4
	5
	6
	7
	8
	9
	10
	Observed Errors



# CLASS PROFILE (3 OF 3)

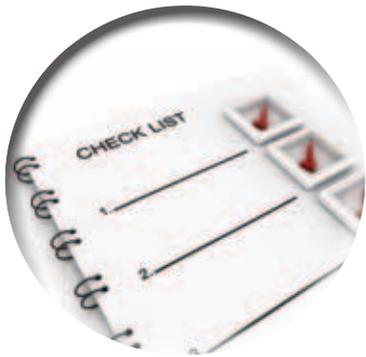
## Mathematical Practice Standards

- MP1:** Make sense of problems and persevere in solving them.
- MP2:** Reason abstractly and quantitatively.
- MP3:** Construct viable arguments and critique the reasoning of others.
- MP4:** Model with mathematics.
- MP5:** Use appropriate tools strategically.
- MP6:** Attend to precision.
- MP7:** Look for and make use of structure.
- MP8:** Look for and express regularity in repeated reasoning.

**Student Name**

**Observations**

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



# A Complete Solution to a Word Problem

includes all of the following ...



A written estimate



All work that you do



An equation (even if you solved it using column form)



A diagram, number line, table, or other representation



The answer to the question in a complete sentence



# What to Do If You Get Stuck



Look at past work times



Look at the charts that are posted



Model the problem using counters or other materials



Sketch a diagram or other representation



Change the numbers to make the problem simpler



Write what you do know



Write down questions to ask later



Check other resources