

Operations with Fractions: Multiplication and Division



AMERICA'S
CHOICE®

SAVVAS
LEARNING COMPANY, INC.

Boston, Massachusetts

Chandler, Arizona

Glenview, Illinois

Upper Saddle River, New Jersey

Online Resources

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Introduction to Math Navigator

Dear Parent/Guardian,

_____ has been selected to participate in Math Navigator! Math Navigator is one of the ways that our school is working to help all students succeed in mathematics. The program gives students the additional time and instruction they need to improve their performance in this important subject.

Your child will be participating in the *Operations with Fractions: Multiplication and Division* module. The main goal of this module is to help students use their understanding of whole numbers to multiply and divide fractions. Students begin the module by dividing whole numbers to find the fraction result. Students also work with rulers and number lines to see fractions as division. Next, students multiply fractions using contextual problems that are carefully sequenced. In the final week of the module, students divide unit fractions by whole numbers and whole numbers by unit fractions. By the end of the module, students are expected to multiply and divide fractions for contextual problems involving mixed operations.

There are a variety of materials students will use with this module: one of them is a set of Study Cards. These cards include mathematical ideas for students to master, game cards, and blank cards that students can customize with concepts that they need to work on. Students are encouraged to use these cards during the lessons, as well as during free time and at home. Please encourage your child to share them with you.

The more enthusiastic you can be about Math Navigator, the more it will help your child. Ask questions each day about what your child learned and how the Math Navigator class was different from your child's regular math class. It is important for you to acknowledge what your child has accomplished both on a day-to-day basis and after completing the Math Navigator module.

We are excited about using Math Navigator with students. Learn more about this special program and how it works by reading the short description that follows. If you have any questions about the program, please do not hesitate to contact us here at school.

How Math Navigator Works

Structure of a Module

Each module contains 20 days of 30- or 45-minute lessons, including a pre-test and post-test. During the 20 days, students have two or three checkpoint lessons that assess their understanding of the concepts in the module.

Frequent Skills Practice

Most lessons include a Show Me session in which students practice and reinforce skills. It is also a time for students to learn strategies and techniques that make computation easier.

Emphasis on Understanding

The lessons are carefully designed to uncover mistakes that result from students misunderstanding something. We call such mistakes *misconceptions*. Misconceptions need to be corrected because they can interfere with new learning. Math Navigator modules do not attempt to reteach everything that students have learned about a topic. Instead, they help students understand the mathematics of the procedures and concepts that they have already learned so that they can correct the misconceptions that are getting in the way of their progress.

Learning to Think Mathematically

Lessons are structured to teach students to think like mathematicians. Students will learn how to ask themselves questions before beginning a problem; to use diagrams, tables, and other methods of representing problems; and to estimate as a way of determining whether their answers are reasonable. Most importantly, they will come to see that mistakes are opportunities for learning, rather than something to hide.

Use a ruler to divide this 6-inch line segment into 2 equal parts.



Measure each of the equal parts. Complete this equation.

$$6 \div 2 =$$

Divide this 6-inch line segment into 6 equal parts. Measure each of the equal parts. Write an equation to express the relationship.



Divide this 6-inch line segment into 3 equal parts. Measure each of the equal parts. Write an equation to express the relationship.



| Problem | |
|---|-------------|
| Tran put 4 photos on each of 6 pages in an album. How many photos did he put in the album? | |
| Diagram | Words |
| | |
| | Calculation |
| | |
| | Answer |
| | |

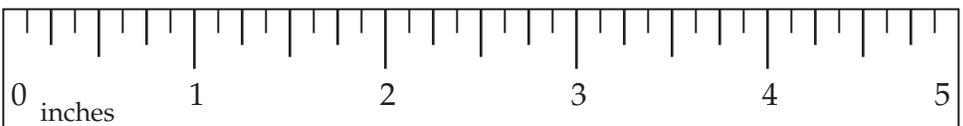
Misconceptions and Errors

| | |
|------------|---|
| F3 | Does not understand how fractions or percents are represented in diagrams/models |
| F15 | Confuses multiplication with addition/subtraction, and adds or subtracts either the numerator and/or denominator |
| F19 | Confuses multiplication of fractions with division and inverts one or both numbers before multiplying |
| F27 | When dividing fractions, divides the second number by the first or inverts both numbers |
| F30 | Does not simplify |
| F31 | Performs the division or multiplication on the numerator and/or the denominator separately |
| F40 | When multiplying fractions, multiplies the numerator of the first fraction by the denominator of the second, and adds the product of the denominator of the first and the numerator of the second |
| F41 | When dividing fractions, multiplies instead |
| F42 | When dividing fractions, divides the numerators and divides the denominators |
| O5 | Does not recognize multiplication situations |
| O6 | Does not recognize division situations |
| O7 | Thinks that when dividing, you always divide by the smaller number |
| O8 | Multiplies or divides incorrectly, or misapplies appropriate procedures for multiplying or dividing |
| O9 | Does not know how to approach the problem—simply uses one of the factors as the answer |
| O10 | Confuses the parts of a division problem, such as dividend and divisor |
| O12 | Thinks that multiplying always makes things bigger |
| O13 | Thinks that dividing always makes things smaller |
| O17 | Estimates incorrectly |
| O18 | Writes an incorrect expression |

F3 Does not understand how fractions or percents are represented in diagrams/models

The student cannot represent or read fractions when present in diagrams—including number lines, area models, and separate figures. Student does not recognize fractions as points on a number line or as division calculations.

example



This line segment is $3\frac{1}{2}$ " long.

F15 Confuses multiplication with addition/subtraction, and adds or subtracts either the numerators and/or denominators

When multiplying/dividing two fractions, the student adds/subtracts the numerators and adds/subtracts the denominators.

example

$$\frac{3}{4} \times \frac{4}{5} = \frac{7}{20}$$

F19 Confuses multiplication of fractions with division and inverts one or both numbers before multiplying

example

Solve.

$$\frac{1}{2} \times \frac{6}{8} = \frac{8}{12}$$

F27 When dividing fractions, divides the second number by the first or inverts both numbers

example

Solve.

$$\frac{1}{2} \div \frac{1}{6} = 12$$

F30 Does not simplify

example

Josh rode his bike $\frac{5}{8}$ of a mile. He rode $\frac{3}{8}$ of a mile more than Anthony.

How far did Anthony ride?

$$\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$$

Anthony rode $\frac{2}{8}$ mile.

F31 Performs the division or multiplication on the numerator and/or the denominator separately

example

Solve.

$$\frac{1}{2} \times \frac{6}{8} = \frac{3}{4}$$

- F40** When multiplying fractions, multiplies the numerator of the first fraction by the denominator of the second, and adds the product of the denominator of the first and the numerator of the second

example

$$\frac{3}{4} \times \frac{4}{5} = 3 \cdot 5 + 4 \cdot 4 = 31$$

- F41** When dividing fractions, multiplies instead

example

$$\frac{3}{4} \div \frac{4}{5} = \frac{12}{20}$$

- F42** When dividing fractions, divides the numerators and divides the denominators

example

$$\frac{3}{4} \div \frac{6}{24} = \frac{2}{6} \text{ or } \frac{1}{3}$$

- O5** Does not recognize multiplication situations

example

Misha bought 11.7 gallons of diesel fuel at \$2.34 per gallon.
How much did he pay for the fuel?

Misha paid \$14.04.

O6 Does not recognize division situations

example Write a calculation that gives the correct answer to the following problem. Angela drives at an average speed of 20.4 miles per hour for 11.6 miles. How many hours does she drive?

11.6×20.4

O7 Thinks that when dividing, you always divide by the smaller number

The student bases all his answers on whole numbers and divides by the smaller number. He does not realize that fractions represent a division.

example Mr. Hakim wants to share 4 sandwiches equally among 8 students. How many sandwiches does each student get?

Each student gets 2 sandwiches.

O8 Multiplies or divides incorrectly, or misapplies appropriate procedures for multiplying or dividing

example Pablo paid \$7.20 for a whole pizza. The pizza was cut into 12 slices. How much did each slice cost?

Each slice cost \$0.50.

O9 Does not know how to approach the problem—simply uses one of the factors as the answer

example What is an expression that would give you an answer of $\frac{1}{8}$?

$\frac{1}{2}$

O10 Confuses the parts of a division problem, such as dividend and divisor

example Mario's family must travel 468 miles to reach his grandparents' house. His father drives 60 miles per hour. To estimate how much time the trip will take, which operation should be used?

You would divide 60 by 500.

O12 Thinks that multiplying always makes things bigger

example Estimate the answer to $30 \times \frac{16}{17}$.

Greater than 30

O13 Thinks that dividing always makes things smaller

example Estimate the answer to $25 \div \frac{3}{4}$.

Less than 25

O17 Estimates incorrectly

example

What is a good estimate for the solution to the following problem?
(Do not solve for the exact answer.)

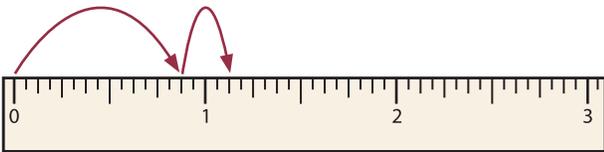
Angela cycles 11.6 miles at a steady speed of 20.4 miles per hour.
How long does she ride?

Angela rides for 0.40 hours.

O18 Writes an incorrect expression

example

Write an expression which matches this diagram.



$\frac{7}{8} - \frac{1}{4}$

Class Profile Instructions

About the Class Profile

Completing an analysis of student work gives you a clear picture of the strategies an individual student is applying to a particular problem or topic in mathematics. Such an analysis is even more powerful when it is applied to the Math Navigator class as a whole.

The Class Profile gives you both. By reading the Class Profile across a row, you can see where each student stands at any point in time. Reading down the columns allows you to see the strengths and needs of the entire class at a glance. By reviewing the Class Profile, you will be able to make decisions that target appropriate instruction to individuals, small groups, and the whole Math Navigator class.

The first pages of the Class Profile provide assessment items related to the content of the module. The last page is based on the mathematical practices from the Common Core State Standards for Mathematics.¹ On this page, record evidence of students using these practices.

Recording Data on the Class Profile

When you see—either through discussion, analysis of student work, or direct observation—that a student understands a concept, still has a misconception, or engages in a mathematical practice, make a note on your Class Profile. As the student's understanding increases, update the Class Profile.

Using the Class Profile

Review the Class Profile periodically during the lesson to help you decide which topics would be most beneficial for your students to focus on during the class discussion. Address topics that most of the students in the Math Navigator group need to learn during the show me, work time, or probing for understanding parts of the lesson. Address topics that only some students are struggling with during partner work or in conferences. If only one or two students need help with a topic, address the topic in an individual conference.

Give a copy of the completed Class Profile to each student's classroom teacher at the end of the module.

¹Common Core State Standards Initiative. 2010. "Common Core State Standards for Mathematics": 6–8. Accessed July 1, 2011. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

CLASS PROFILE (1 OF 3)

| Concepts | |
|---------------------|---|
| Student Name | |
| 1 | C1: Interprets a fraction as division of the numerator by the denominator ($a/b = a \div b$) |
| 2 | C2: Accounts for remainders in word problems involving division of whole numbers in the form of fractions or mixed numbers appropriately |
| 3 | C3: Explains the product $(a/b) \times q$ as a parts of a partition of q into b equal parts or as the result of a sequence of operations $a \times q \div b$ |
| 4 | C4: Recognizes and explains situations in which multiplication and division of fractions are needed |
| 5 | C5: Represents fraction products using visual models (linear models, rectangular area models, and/or set models) |
| 6 | C6: Explains why multiplying a given number by a fraction $>$, $<$, or $= 1$ results in a product that is greater than, less than, or equal to the number |
| 7 | C7: Solves real-world problems involving multiplication of fractions and mixed numbers |
| 8 | C8: Clarifies the meaning of division of a unit fraction by a whole number or a whole number by a unit fraction using visual models or verbal explanations |
| 9 | C9: Solves real-world problems involving division of unit fractions by whole numbers and whole numbers by unit fractions |
| 10 | Observed Errors |

CLASS PROFILE (3 OF 3)

Mathematical Practice Standards

- MP1:** Make sense of problems and persevere in solving them.
- MP2:** Reason abstractly and quantitatively.
- MP3:** Construct viable arguments and critique the reasoning of others.
- MP4:** Model with mathematics.
- MP5:** Use appropriate tools strategically.
- MP6:** Attend to precision.
- MP7:** Look for and make use of structure.
- MP8:** Look for and express regularity in repeated reasoning.

Student Name

Observations

| | |
|----|--|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |



A Complete Solution to a Word Problem

includes all of the following ...



A written estimate



All work that you do



An equation (even if you solved it using column form)



A diagram, number line, table, or other representation



The answer to the question in a complete sentence



What to Do If You Get Stuck



Look at past work times



Look at the charts that are posted



Model the problem using counters or other materials



Sketch a diagram or other representation



Change the numbers to make the problem simpler



Write what you do know



Write down questions to ask later



Check other resources