

Operations with Fractions: Addition and Subtraction



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CHOICE.

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Online Resources

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Introduction to Math Navigator

Dear Parent/Guardian,

_____ has been selected to participate in Math Navigator! Math Navigator is one of the ways that our school is working to help all students succeed in mathematics. The program gives students the additional time and instruction they need to improve their performance in this important subject.

Your child will be participating in the *Operations with Fractions: Addition and Subtraction* module. The main goal of this module is to help students use their understanding of whole numbers to add and subtract fractions. During the module, students will estimate and find exact answers using benchmark fractions, area models, rulers, and number lines as scaffolds. By the end of the module, students will understand what it means to give answers in simplest form and why that is the conventional way of expressing fractional amounts.

There are a variety of materials students will use with this module: one of them is a set of Study Cards. These cards include mathematical ideas for students to master, game cards, and blank cards that students can customize with concepts that they need to work on. Students are encouraged to use these cards during the lessons, as well as during free time and at home. Please encourage your child to share them with you.

The more enthusiastic you can be about Math Navigator, the more it will help your child. Ask questions each day about what your child learned and how the Math Navigator class was different from your child's regular math class. It is important for you to acknowledge what your child has accomplished both on a day-to-day basis and after completing the Math Navigator module.

We are excited about using Math Navigator with students. Learn more about this special program and how it works by reading the short description that follows. If you have any questions about the program, please do not hesitate to contact us here at school.

How Math Navigator Works

Structure of a Module

Each module contains 20 days of 30- or 45-minute lessons, including a pre-test and post-test. During the 20 days, students have two or three checkpoint lessons that assess their understanding of the concepts in the module.

Frequent Skills Practice

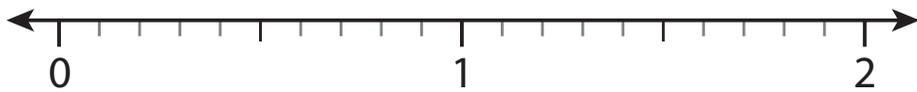
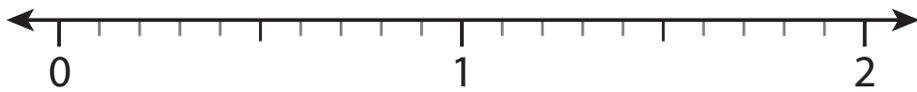
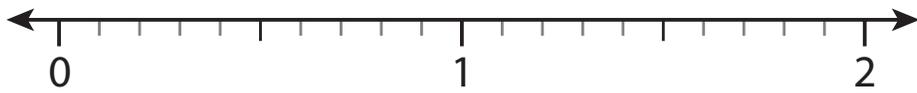
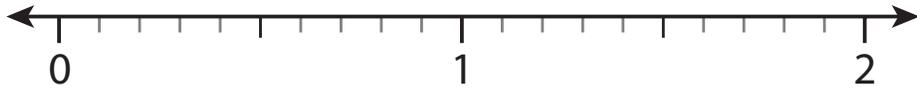
Most lessons include a Show Me session in which students practice and reinforce skills. It is also a time for students to learn strategies and techniques that make computation easier.

Emphasis on Understanding

The lessons are carefully designed to uncover mistakes that result from students misunderstanding something. We call such mistakes *misconceptions*. Misconceptions need to be corrected because they can interfere with new learning. Math Navigator modules do not attempt to reteach everything that students have learned about a topic. Instead, they help students understand the mathematics of the procedures and concepts that they have already learned so that they can correct the misconceptions that are getting in the way of their progress.

Learning to Think Mathematically

Lessons are structured to teach students to think like mathematicians. Students will learn how to ask themselves questions before beginning a problem; to use diagrams, tables, and other methods of representing problems; and to estimate as a way of determining whether their answers are reasonable. Most importantly, they will come to see that mistakes are opportunities for learning, rather than something to hide.



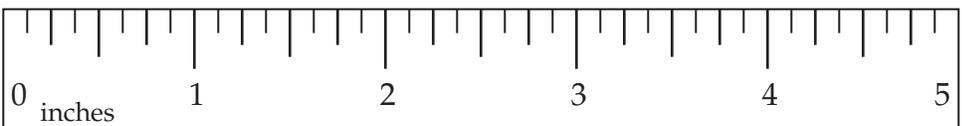
Misconceptions and Errors

F3	Does not understand how fractions or percents are represented in diagrams/models
F7	Understands that an operation is needed to make the fractions equivalent but applies the wrong one
F9	When converting to or from a mixed number, performs the wrong operation when dealing with the numerator
F13	Confuses addition and subtraction of fractions with multiplication; adds or subtracts the numerators and denominators
F17	When adding or subtracting fractions, adds or subtracts the numerators (without converting to an equivalent fraction) and places the result over a common denominator
F18	Adds or subtracts a whole number and an improper fraction by dealing with the whole number as a number in the numerator
F28	Does not understand how to use benchmark numbers
F30	Does not simplify
F35	Thinks that inverse fractions are equivalent
F39	When subtracting, subtracts the smaller whole number from the larger and the smaller fraction from the larger
F43	Does not correctly calculate an operation with fractions
F44	Thinking that doubling the size of the denominator doubles the size of the fraction
F45	When adding or subtracting, adds the numerators (or the denominators) and gives that as the answer
O3	Does not recognize an addition situation
O4	Does not recognize a subtraction situation
O12	Thinks that multiplying always makes things bigger
O13	Thinks that dividing always makes things smaller
O17	Estimates incorrectly
O18	Writes an incorrect expression

F3 Does not understand how fractions or percents are represented in diagrams/models

The student cannot represent or read fractions when present in diagrams—including number lines, area models, and separate figures. Student does not recognize fractions as points on a number line or as division calculations.

example



This line segment is $3\frac{1}{2}$ " long.

F7 Understands that an operation is needed to make the fractions equivalent, but applies the wrong one

The student knows she needs do something to make equivalent fractions but gets confused and does the wrong operation. The student misapplies additive ideas when finding equivalent fractions.

example

Are $\frac{2}{3}$ and $\frac{4}{5}$ equivalent fractions?

Yes, because you can add 2 to the numerator and denominator: $\frac{2+2}{3+2}$ to get $\frac{4}{5}$.

F9 When converting to or from a mixed number, performs the wrong operation when dealing with the numerator

example

Change $2\frac{3}{5}$ into an improper fraction.

$$2\frac{3}{5} = \frac{8}{5}$$

F13 Confuses with multiplication and adds or subtracts both numerator and denominator

When adding/subtracting two fractions, the student adds/subtracts the numerators and adds/subtracts the denominators.

example

$$\frac{1}{3} + \frac{3}{5} = \frac{4}{8}$$

F17 Adds or subtracts numerators (without converting to an equivalent fraction) and places over a common denominator

The student does not understand that the fraction must have the same denominator before he can add or subtract.

example

$$\frac{2}{5} + \frac{4}{9} =$$

$$\frac{2}{5} + \frac{4}{9} = \frac{6}{45}$$

F18 Adds or subtracts a whole number and an improper fraction by dealing with the whole number as a number in the numerator

example

$$2\frac{1}{4} - 1\frac{4}{9} =$$

$$2\frac{1}{4} - 1\frac{4}{9} = \frac{7}{4}$$

F28 Does not understand how to use benchmark numbers

The student does not use benchmark fractions in trying to solve problems, but instead does a rote procedural approach.

example

Compare $\frac{7}{11}$ and $\frac{5}{13}$.

The student starts cutting fraction pieces, resorts to guessing, or performs difficult computations (such as finding the decimal equivalents or common denominators) instead of comparing both numbers to one-half.

F30 Does not simplify

example

Josh rode his bike $\frac{5}{8}$ of a mile. He rode $\frac{3}{8}$ of a mile more than Anthony.

How far did Anthony ride?

$\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$ Anthony rode $\frac{2}{8}$ mile.

F35 Thinks that inverse fractions are equivalent

The student confuses the idea of equivalence with the idea of inverse.

example

Which of these fractions are equal: $\frac{2}{3}, \frac{4}{6}, \frac{3}{2}$

$\frac{2}{3} = \frac{3}{2}$

F39 When subtracting, subtracts the smaller whole number from the larger and the smaller fraction from the larger

example

$$3\frac{2}{5} - 1\frac{4}{5} = 2\frac{2}{5}$$

F43 Does not correctly calculate an operation with fractions

example

$$\frac{2}{3} + \frac{3}{4} = \frac{5}{12} \qquad \frac{1}{2} + \frac{4}{5} = \frac{5}{7}$$

F44 Thinking that doubling the size of the denominator doubles the size of the fraction

example

Which is greater, $\frac{1}{12}$ or $\frac{1}{24}$? $\frac{1}{24}$

F45 When adding or subtracting, adds the numerators (or the denominators) and gives that as the answer

example

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3$$

O3 Does not recognize an addition situation

The student overspecializes during the learning process so that he recognizes some situations as addition but fails to classify other addition situations appropriately.

example

The temperature was 47° at 8 am, which was 12° cooler than it is now. What is the temperature now?

The temperature now is 35° . $47 - 12 = 35$

O4 Does not recognize a subtraction situation

The student overspecializes during the learning process so that she recognizes some subtraction situations as subtraction but fails to classify other subtraction situations appropriately.

example

If there are 7 birds in a bush and 3 fly away, how many are left?

$7 - 3 = 4$. 4 birds are left.

Tom and Megan have 24 books altogether. Megan has 12 books. How many books does Tom have?

$24 + 12 = 36$. Tom has 36 books.

O12 Thinks that multiplying always makes things bigger

example

Estimate the answer to $30 \times \frac{16}{17}$.

Greater than 30

O13 Thinks that dividing always makes things smaller

example

Estimate the answer to $25 \div \frac{3}{4}$.

Less than 25

O17 Estimates incorrectly

example

What is a good estimate for the solution to the following problem?
(Do not solve for the exact answer.)

Angela cycles 11.6 miles at a steady speed of 20.4 miles per hour.
How long does she ride?

Angela rides for 0.40 hours.

O18 Writes an incorrect expression

example

Write an expression which matches this diagram.



$\frac{7}{8} - \frac{1}{4}$

Class Profile Instructions

About the Class Profile

Completing an analysis of student work gives you a clear picture of the strategies an individual student is applying to a particular problem or topic in mathematics. Such an analysis is even more powerful when it is applied to the Math Navigator class as a whole.

The Class Profile gives you both. By reading the Class Profile across a row, you can see where each student stands at any point in time. Reading down the columns allows you to see the strengths and needs of the entire class at a glance. By reviewing the Class Profile, you will be able to make decisions that target appropriate instruction to individuals, small groups, and the whole Math Navigator class.

The first pages of the Class Profile provide assessment items related to the content of the module. The last page is based on the mathematical practices from the Common Core State Standards for Mathematics.¹ On this page, record evidence of students using these practices.

Recording Data on the Class Profile

When you see—either through discussion, analysis of student work, or direct observation—that a student understands a concept, still has a misconception, or engages in a mathematical practice, make a note on your Class Profile. As the student’s understanding increases, update the Class Profile.

Using the Class Profile

Review the Class Profile periodically during the lesson to help you decide which topics would be most beneficial for your students to focus on during the class discussion. Address topics that most of the students in the Math Navigator group need to learn during the show me, work time, or probing for understanding parts of the lesson. Address topics that only some students are struggling with during partner work or in conferences. If only one or two students need help with a topic, address the topic in an individual conference.

Give a copy of the completed Class Profile to each student’s classroom teacher at the end of the module.

¹Common Core State Standards Initiative. 2010. “Common Core State Standards for Mathematics”: 6–8. Accessed July 1, 2011. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

CLASS PROFILE (2 OF 2)

Mathematical Practice Standards	
<p>MP1: Make sense of problems and persevere in solving them.</p> <p>MP2: Reason abstractly and quantitatively.</p> <p>MP3: Construct viable arguments and critique the reasoning of others.</p> <p>MP4: Model with mathematics.</p>	<p>MP5: Use appropriate tools strategically.</p> <p>MP6: Attend to precision.</p> <p>MP7: Look for and make use of structure.</p> <p>MP8: Look for and express regularity in repeated reasoning.</p>
Student Name	Observations
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



A Complete Solution to a Word Problem

includes all of the following ...



A written estimate



All work that you do



An equation (even if you solved it using column form)



A diagram, number line, table, or other representation



The answer to the question in a complete sentence



What to Do If You Get Stuck



Look at past work times



Look at the charts that are posted



Model the problem using counters or other materials



Sketch a diagram or other representation



Change the numbers to make the problem simpler



Write what you do know



Write down questions to ask later



Check other resources