

# Expressions



AMERICA'S  
CHOICE.

**SAVVAS**  
LEARNING COMPANY

Boston, Massachusetts

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Upper Saddle River, New Jersey

Online Resources

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## Introduction to Math Navigator

Dear Parent/Guardian,

\_\_\_\_\_ has been selected to participate in Math Navigator! Math Navigator is one of the ways that our school is working to help all students succeed in mathematics. The program gives students the additional time and instruction they need to improve their performance in this important subject.

Your child will be participating in the *Expressions* module. The main goal of this module is to give students an overview of how to create and interpret expressions. Students will simplify numerical expressions using parentheses, the distributive property, and order of operations. They will use this knowledge to learn how to manipulate expressions with variables both by translating between words and algebraic expressions and by connecting expressions to corresponding area models and tables. Students will simplify algebraic expressions by combining like terms. In the fourth week, students will use their knowledge of expressions to solve word problems that require writing first-degree equations.

There are a variety of materials students will use with this module: one of them is a set of Study Cards. These cards include mathematical ideas for students to master, game cards, and blank cards that students can customize with concepts that they need to work on. Students are encouraged to use these cards during the lessons, as well as during free time and at home. Please encourage your child to share them with you.

The more enthusiastic you can be about Math Navigator, the more it will help your child. Ask questions each day about what your child learned and how the Math Navigator class was different from your child's regular math class. It is important for you to acknowledge what your child has accomplished both on a day-to-day basis and after completing the Math Navigator module.

We are excited about using Math Navigator with students. Learn more about this special program and how it works by reading the short description that follows. If you have any questions about the program, please do not hesitate to contact us here at school.

### How Math Navigator Works

#### Structure of a Module

Each module contains 20 days of 30- or 45-minute lessons, including a pre-test and post-test. During the 20 days, students have two or three checkpoint lessons that assess their understanding of the concepts in the module.

#### Frequent Skills Practice

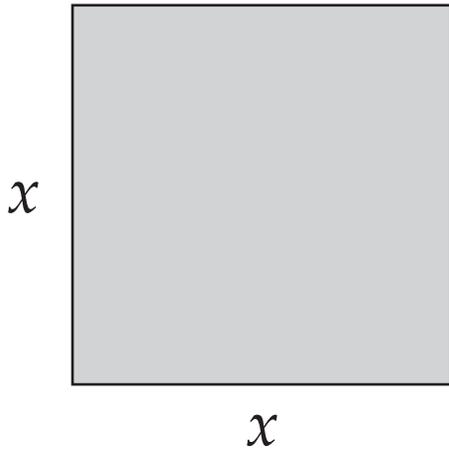
Most lessons include a Show Me session in which students practice and reinforce skills. It is also a time for students to learn strategies and techniques that make computation easier.

#### Emphasis on Understanding

The lessons are carefully designed to uncover mistakes that result from students misunderstanding something. We call such mistakes *misconceptions*. Misconceptions need to be corrected because they can interfere with new learning. Math Navigator modules do not attempt to reteach everything that students have learned about a topic. Instead, they help students understand the mathematics of the procedures and concepts that they have already learned so that they can correct the misconceptions that are getting in the way of their progress.

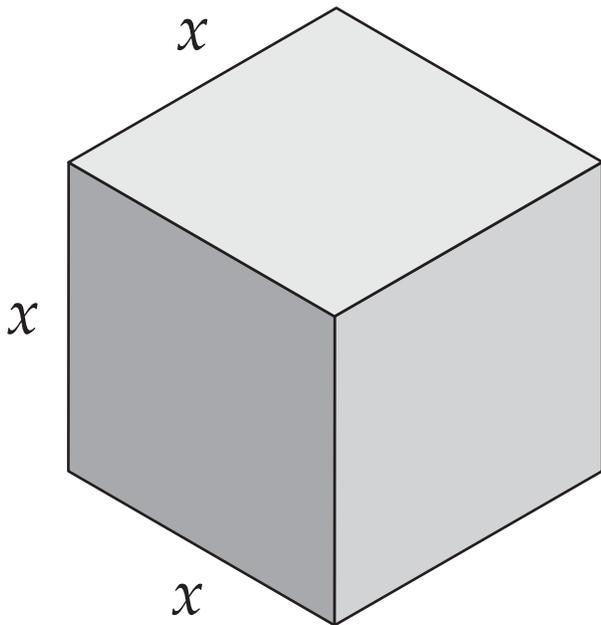
#### Learning to Think Mathematically

Lessons are structured to teach students to think like mathematicians. Students will learn how to ask themselves questions before beginning a problem; to use diagrams, tables, and other methods of representing problems; and to estimate as a way of determining whether their answers are reasonable. Most importantly, they will come to see that mistakes are opportunities for learning, rather than something to hide.



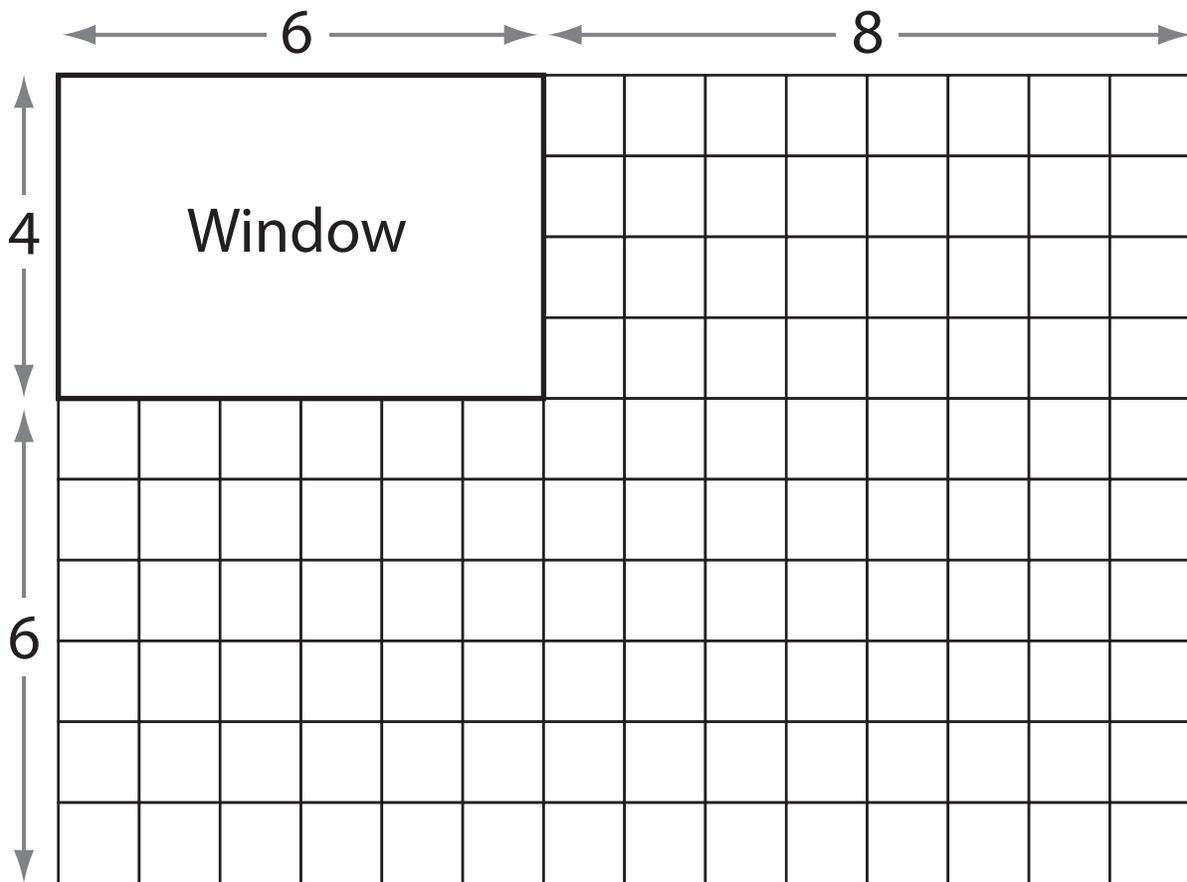
**Square**

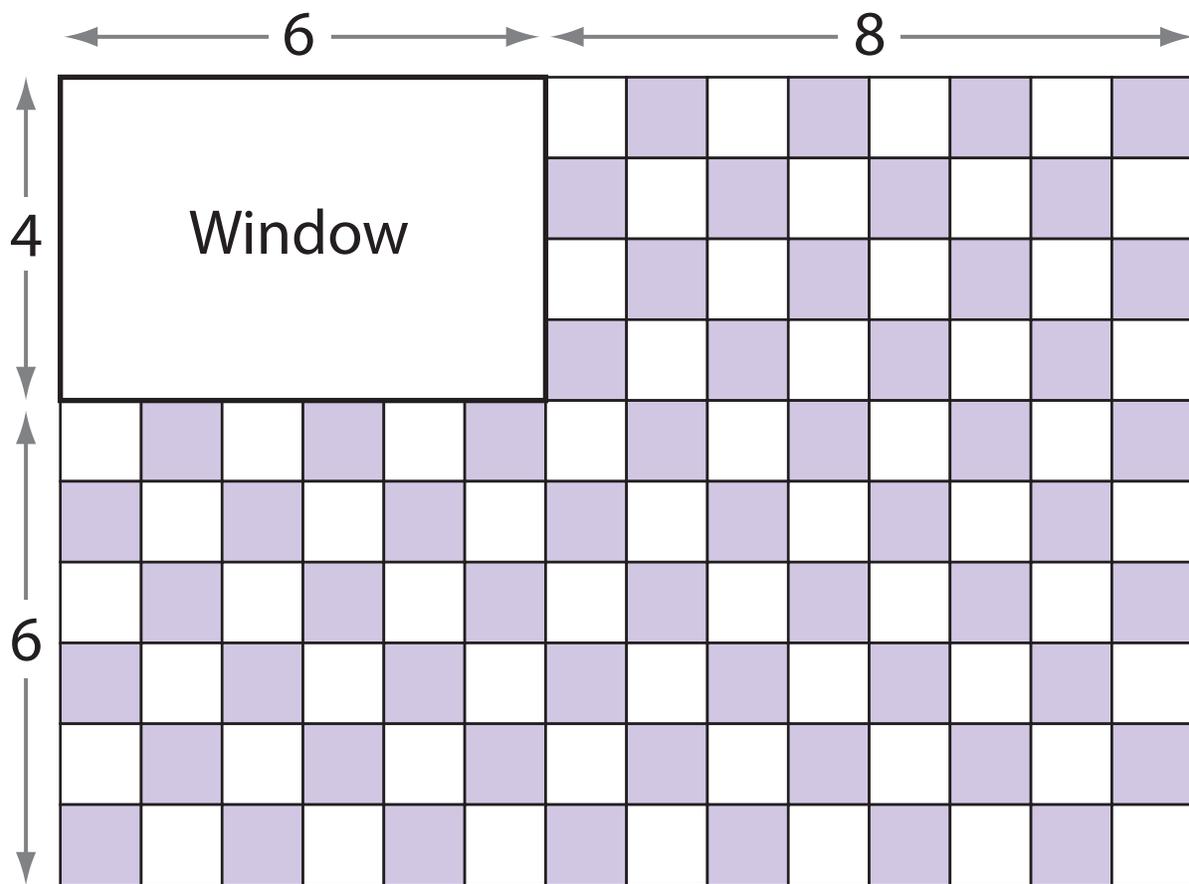
$$\text{Area} = x \cdot x = x^2$$



**Cube**

$$\text{Volume} = x \cdot x \cdot x = x^3$$





## Misconceptions and Errors

<b>A16</b>	Does not understand conventions for writing expressions
<b>E1</b>	Fails to apply or misapplies order of operations principles
<b>E2</b>	Thinks that a number raised to a power is the same as the number multiplied by a power
<b>E5</b>	Is unable to determine, express, or apply the rule that matches a given table or graph
<b>E6</b>	Is unable to transfer between verbal, visual, and/or numerical representations of expressions or equations
<b>E7</b>	Reverses the order of the expression
<b>E10</b>	Incorrectly evaluates an expression using substitution
<b>E11</b>	Does not understand adding like terms
<b>E12</b>	Does not know how to simplify an expression
<b>M7</b>	Confuses the formulas for area and perimeter
<b>M9</b>	Uses the formula for the area of a rectangle for a nonrectangular shape
<b>M17</b>	Does not know formulas for area and perimeter
<b>M18</b>	Does not understand or ignores unit conversion
<b>O2</b>	Adds or subtracts incorrectly—fails to apply or misapplies appropriate procedures for adding or subtracting
<b>O3</b>	Does not recognize an addition situation
<b>O4</b>	Does not recognize a subtraction situation
<b>O5</b>	Does not recognize a multiplication situation
<b>O6</b>	Does not recognize a division situation
<b>O8</b>	Multiplies or divides incorrectly or misapplies appropriate procedures for multiplying or dividing
<b>O14</b>	Does not recognize or misapplies the commutative property
<b>O24</b>	Does not recognize or misapplies the distributive property of multiplication

**A16 Does not understand conventions for writing expressions**

**example**

$x \cdot x$  rather than  $x^2$

$x \cdot 5 = 1$  rather than  $5x = 1$

**E1 Fails to apply or misapplies order of operations principles**

The student evaluates problems from left to right no matter what the operations are.

**example**

$20 + 6 \cdot 3 = 78$

$(2 \cdot 9) + 22 - 20 - (8 \cdot 2) = 24$

**example**

Work with a partner.

1. Jong handed in the following homework. For each problem, correct any wrong answers and explain what mistake(s) he made.

10. When you add 7 to an unknown number and then multiply by 4, it equals 40. What is the unknown number?

$x + 7 \cdot 4 = 40$      $(x+7) \cdot 4 = 40$      $x+7=10$

$x + 28 = 40$      $\frac{4}{4} \frac{40}{4}$      $\frac{-7}{-7} \frac{10}{-7}$

$x + 28 - 28 = 40 - 28$      $x = 33$

$x = 12$

**E2 Thinks that a number raised to a power is the same as the number multiplied by a power**

The student multiplies by an exponent rather than multiplying the expression by itself.

**example**

$6^2 = 6 \cdot 2$

$4^3 - (39 \div 3) + 7 = 6$

**E5 Is unable to determine, express, or apply the rule that matches a given table or graph**

**example**

What numbers would complete the table to match the rule “Add 3 to each number, then multiply by 5”?

$x$	5	2	12	3	43
Answers	28	13	63	18	218

**E6 Is unable to transfer between verbal, visual, and/or numerical representations of expressions or equations**

**example**

Add 5 to  $x$  and multiply the quantity by 2.

$$5 + x \cdot 2$$

**E7 Reverses the order of the expression**

**example**

Find the algebraic expression for “Subtract 5 from a number, then divide by 4.”

$$\frac{5 - x}{4}$$

**E10 Incorrectly evaluates an expression using substitution**

**example**

Substitute  $y = 2$  in  $y + 1 = y - 1$ .

$1 = 1$ , the equation is true.

**E11 Does not understand adding like terms**

example

$$10 + 2x = 12x$$

$$10x^2 + 2x = 12x^2$$

**E12 Does not know how to simplify an expression**

example

$$4 + 2x + 6 + x = 10 + 2x + x$$

**M7 Confuses the formulas for area and perimeter**

The student interprets all “fence” problems as perimeter, even if the problem talks about the size of the garden that the fence encloses, or the student interprets all “wall painting” problems as area, even if the problem talks about the length of a striped border that is painted around the room.

example

Find the area of a rectangle with dimensions of 12 cm × 4 cm.

$$12 + 4 + 12 + 4 = 32 \text{ cm}$$

example

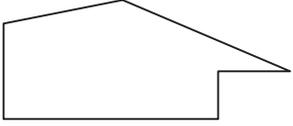
Find the perimeter of a rectangle with dimensions of 8 inches × 7 inches.

$$8 \times 7 = 56 \text{ inches}$$

**M9 Uses the formula for the area of a rectangle for a nonrectangular shape**

**example**

The length is 7 cm.  
 The width is 5 cm.  
 The area is  $7 \cdot 5 = 35$  sq cm.



**M17 Does not know formulas for area and perimeter**

**example**

Sue wants to build 2 different rectangular gardens, each having an area of 12 square meters. Write a set of dimensions Sue could use to build her two gardens.

6 meters by 6 meters and 12 meters by 1 meter

**M18 Does not understand or ignores unit conversion**

**example**

Katie spent 2 hours cleaning her room. Then she spent 75 minutes watching a video. How much time did Katie spend cleaning her room and watching a video?

Katie spent 2.75 hours watching the video and cleaning her room.

**O2 Adds or subtracts incorrectly—fails to apply or misapplies appropriate procedures for adding or subtracting**

**example**

$32 - 12 \div 2 = 10$

What numbers would complete the table to match the expression  $x - 6$ ?

$x$	13	7	6	21	12
Answers	7	1	0	16	6

**O3 Does not recognize an addition situation**

**example**

$20 + (5 \cdot 3) = 160$

Marita makes \$1.50 more per hour than her brother Pablo at their summer jobs. Pablo makes  $x$  dollars per hour. Which expression shows how much Marita makes per hour?

$(x - 1.50)$  dollars

**O4 Does not recognize a subtraction situation**

**example**

$48 \div (4 - 1) = 9.6$

**O5 Does not recognize a multiplication situation**

**example**

$$196 \div 14 \cdot 3 = 17$$

Which expression shows the cost of renting a bicycle at \$1.50 per hour for  $x$  hours?

$x + 1.5$  dollars

**O6 Does not recognize a division situation**

**example**

$$54 \div (11 - 2) = 45$$

Write a situation that corresponds to this equation:  $y = \frac{12}{x}$ .

Angela is twelve years older than Carlo. Angela is  $x$  years old; Carlo is  $y$  years old.

**O8 Multiplies or divides incorrectly or misapplies appropriate procedures for multiplying or dividing**

The student divides the whole expression by the denominator rather than just the part that is the fraction, or does not divide the whole expression.

**example**

$196 \div 14 \cdot 3 = 39$

What number makes this equation true?

$10 \div 5 = 25 \div 10$

**example**

Equation 3

Finish  $\frac{x+7}{5} = 25$

- 2

$\frac{x+9}{5} = 27$

$\cdot 3$

$\frac{x}{5} + 3 = 9$

+ 3

$\frac{x}{5} = 6$

$\div 5$

Start  $x = 30$

**O14 Does not recognize or misapplies the commutative property**

The student thinks subtraction is commutative.

**example**

$$5 - 3 = 3 - 5$$

The student thinks division is commutative.

**example**

$$5 \div 3 = 3 \div 5$$

**O24 Does not recognize or misapplies the distributive property of multiplication**

The student does not distribute multiplication to all terms in the parentheses.

**example**

$$2(x + 6) = 2x + 6$$

The student distributes multiplication by a negative term (or subtraction) to only the first term in an expression.

**example**

$$x - 2(x + 6) = x - 2x + 12 = -x + 12$$

## Class Profile Instructions

### About the Class Profile

Completing an analysis of student work gives you a clear picture of the strategies an individual student is applying to a particular problem or topic in mathematics. Such an analysis is even more powerful when it is applied to the Math Navigator class as a whole.

The Class Profile gives you both. By reading the Class Profile across a row, you can see where each student stands at any point in time. Reading down the columns allows you to see the strengths and needs of the entire class at a glance. By reviewing the Class Profile, you will be able to make decisions that target appropriate instruction to individuals, small groups, and the whole Math Navigator class.

The first pages of the Class Profile provide assessment items related to the content of the module. The last page is based on the mathematical practices from the Common Core State Standards for Mathematics.<sup>1</sup> On this page, record evidence of students using these practices.

### Recording Data on the Class Profile

When you see—either through discussion, analysis of student work, or direct observation—that a student understands a concept, still has a misconception, or engages in a mathematical practice, make a note on your Class Profile. As the student’s understanding increases, update the Class Profile.

### Using the Class Profile

Review the Class Profile periodically during the lesson to help you decide which topics would be most beneficial for your students to focus on during the class discussion. Address topics that most of the students in the Math Navigator group need to learn during the show me, work time, or probing for understanding parts of the lesson. Address topics that only some students are struggling with during partner work or in conferences. If only one or two students need help with a topic, address the topic in an individual conference.

Give a copy of the completed Class Profile to each student’s classroom teacher at the end of the module.

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<sup>1</sup>Common Core State Standards Initiative. 2010. “Common Core State Standards for Mathematics”: 6–8. Accessed July 1, 2011. [http://www.corestandards.org/assets/CCSSI\\_Math%20Standards.pdf](http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf).

# CLASS PROFILE (1 OF 3)

Student Name	C1: Applies the conventional order of operations when evaluating expressions	C2: Recognizes that addition and multiplication are commutative, but subtraction and division are not	C3: Applies the distributive properties of multiplication and division over addition and subtraction (expansion of parentheses)	C4: Recognizes when two expressions are equivalent	C5: Demonstrates understanding of whole-number exponents when writing and evaluating numerical expressions	C6: Recognizes expressions that record operations with numbers and with letters standing for numbers	Observed Errors
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							



# CLASS PROFILE (3 OF 3)

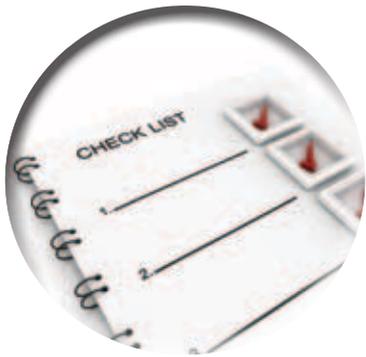
## Mathematical Practice Standards

- MP1:** Make sense of problems and persevere in solving them.
- MP2:** Reason abstractly and quantitatively.
- MP3:** Construct viable arguments and critique the reasoning of others.
- MP4:** Model with mathematics.
- MP5:** Use appropriate tools strategically.
- MP6:** Attend to precision.
- MP7:** Look for and make use of structure.
- MP8:** Look for and express regularity in repeated reasoning.

**Student Name**

**Observations**

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



# A Complete Solution to a Word Problem

includes all of the following ...



A written estimate



All work that you do



An equation (even if you solved it using column form)



A diagram, number line, table, or other representation



The answer to the question in a complete sentence



# What to Do If You Get Stuck



Look at past work times



Look at the charts that are posted



Model the problem using counters or other materials



Sketch a diagram or other representation



Change the numbers to make the problem simpler



Write what you do know



Write down questions to ask later



Check other resources