

Decimals and Powers of Ten



AMERICA'S
CHOICE.

SAVVAS
LEARNING COMPANY

Boston, Massachusetts

Chandler, Arizona

Glenview, Illinois

Upper Saddle River, New Jersey

Online Resources

This work is protected by United States copyright laws and is provided *solely for the use of teachers and administrators* in teaching courses and assessing student learning in their classes and schools. Dissemination or sale of *any* part of this work (including on the World Wide Web) will destroy the integrity of the work and is *not* permitted.

Copyright © 2012 Savvas Education, Inc., or its affiliate(s). All Rights Reserved. Printed in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. The publisher hereby grants permission to reproduce these pages, in part or in whole, for classroom use only, the number not to exceed the number of students in each class. Notice of copyright must appear on all copies. For information regarding permissions, write to Savvas Curriculum Group Rights & Permissions, One Lake Street, Upper Saddle River, New Jersey 07458.

America's Choice, the America's Choice A logo, Math Navigator, the Savvas logo, and the Savvas logo are trademarks, in the U.S. and/or other countries, of Savvas Education, Inc. or its affiliate(s).

ISBN: 978-0-66364-320-3
1 2 3 4 5 6 7 8 9 10 16 15 14 13 12

Lesson 1	Letter to Parents	1
Lesson 4	Number Line Problems	3
Lesson 7	Number Line	4
Lesson 7	Comparing Sporting Events	5
Lesson 13	Place Value Table	6
Lesson 16	Number Lines from 0 to 2	7
	Misconceptions	8
	Class Profile	17
	A Complete Solution to a Word Problem	21
	What to Do If You Get Stuck	22

Introduction to Math Navigator

Dear Parent/Guardian,

_____ has been selected to participate in Math Navigator! Math Navigator is one of the ways that our school is working to help all students succeed in mathematics. The program gives students the additional time and instruction they need to improve their performance in this important subject.

Your child will be participating in the *Decimals and Powers of Ten* module. The main goal of this module is to strengthen students' understanding of decimal place value through identifying and addressing common misconceptions. The module focuses on misconceptions related to adding and subtracting decimals, recognizing the role of zero in numbers, comparing decimals, multiplying and dividing by powers of ten, adding and subtracting currency, and understanding the denseness of numbers.

There are a variety of materials students will use with this module: one of them is a set of Study Cards. These cards include mathematical ideas for students to master, game cards, and blank cards that students can customize with concepts that they need to work on. Students are encouraged to use these cards during the lessons, as well as during free time and at home. Please encourage your child to share them with you.

The more enthusiastic you can be about Math Navigator, the more it will help your child. Ask questions each day about what your child learned and how the Math Navigator class was different from your child's regular math class. It is important for you to acknowledge what your child has accomplished both on a day-to-day basis and after completing the Math Navigator module.

We are excited about using Math Navigator with students. Learn more about this special program and how it works by reading the short description that follows. If you have any questions about the program, please do not hesitate to contact us here at school.

How Math Navigator Works

Structure of a Module

Each module contains 20 days of 30- or 45-minute lessons, including a pre-test and post-test. During the 20 days, students have two or three checkpoint lessons that assess their understanding of the concepts in the module.

Frequent Skills Practice

Most lessons include a Show Me session in which students practice and reinforce skills. It is also a time for students to learn strategies and techniques that make computation easier.

Emphasis on Understanding

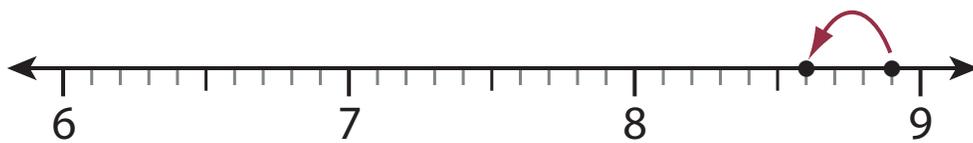
The lessons are carefully designed to uncover mistakes that result from students misunderstanding something. We call such mistakes *misconceptions*. Misconceptions need to be corrected because they can interfere with new learning. Math Navigator modules do not attempt to reteach everything that students have learned about a topic. Instead, they help students understand the mathematics of the procedures and concepts that they have already learned so that they can correct the misconceptions that are getting in the way of their progress.

Learning to Think Mathematically

Lessons are structured to teach students to think like mathematicians. Students will learn how to ask themselves questions before beginning a problem; to use diagrams, tables, and other methods of representing problems; and to estimate as a way of determining whether their answers are reasonable. Most importantly, they will come to see that mistakes are opportunities for learning, rather than something to hide.



Add on 0.4 each time, starting at 5.1.

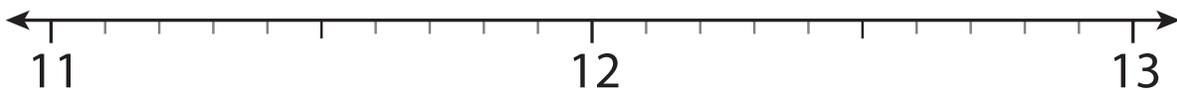


Subtract 0.3 each time, starting at 8.9.

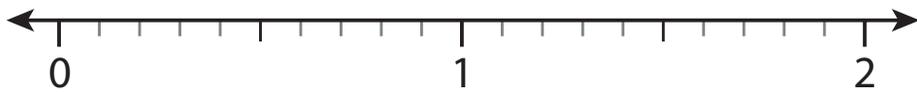
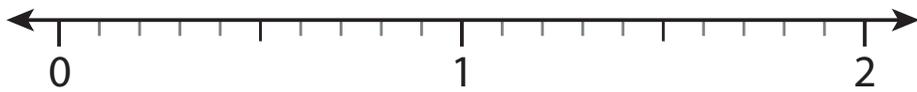
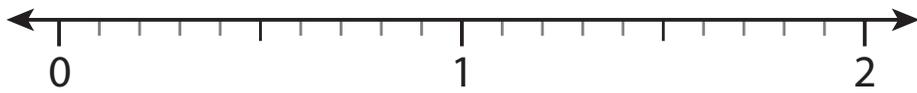
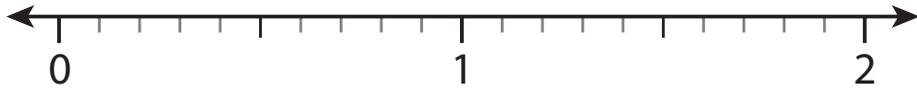


Women's 100-Meter Sprint	
Competitor	Time (seconds)
Angela	12.2
Bettina	12
Carla	12.15
Dolores	11.76
Eiko	11.6
Faith	12.08
Gabby	12.35
Hannah	11.9

Women's 100-Meter Sprint	
Competitor	Time (seconds)



0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000



Misconceptions and Errors

D1	Confuses place values in the decimal part of the number
D2	Confuses place values between the decimal part and the integer part of the number
D3	Reads decimals as whole numbers without designating the place value
D5	Mixes decimals and fractions when reading decimal numbers
D7	Treats the decimal point as a separating point
D11	Does not line up the decimal points before adding or subtracting
D13	Multiplies or divides incorrectly by a power of ten, or uses scientific notation incorrectly
D15	Compares decimals by examining lengths—thinks that shorter lengths mean larger numbers (as with fraction denominators)
D16	Compares decimals by examining lengths—thinks that longer lengths mean larger numbers (as in whole numbers)
D17	Orders decimals based on the first or last digit of the number or the decimal part
D21	Misapplies knowledge of fractions
D26	Does not recognize the “denseness” of decimals, due to a lack of understanding of place value
G3	Rounds incorrectly
NL1	Counts the number of tick marks as starting with 0 and ignores the range of the number line
NL2	Counts the number of tick marks instead of the number of intervals
NL7	Fails to interpret the scale on a graph or number line
O1	Does not use the correct operation
PV1	Does not understand or misinterprets the role of zero as a placeholder

D1 Confuses place values in the decimal part of the number

example Find the value of the digit 9 in this number. 471.3291
The value is 9 hundredths.

D2 Confuses place values between the decimal part and the integer part of the number

example Find the value of the digit 8 in this number. 145.481
The value is 8 hundreds.

What number do you get if you add four-tenths to 276.219?
316.219

D3 Reads decimals as whole numbers without designating the place value

The student verbalizes decimals as whole numbers without the place value designated.

example What is the value of 0.10? *Point ten*

Read this number. 45.7 *Four fifty-seven*

Read this number. 25.37
Two thousand five hundred thirty-seven

D5 Mixes decimals and fractions when reading decimal numbers

example Jan is 1.8 meters tall. There are one hundred centimeters in a meter. Write Jan's height a different way.
Jan is one-eighth of a meter tall.

Convert $\frac{1}{2}$ to a decimal. *1.2*

D7 Treats the decimal point as a separating point

The student uses the decimal point when dealing with money to separate the dollars from the cents, or when dealing with measurement to separate different units. When adding and subtracting, the students does not carry over the decimal point. When multiplying and dividing, the student calculates the whole number separately from the decimal part.

example

100 cents = $\$0.100$

2.8 meters = 2 meters 8 centimeters

example

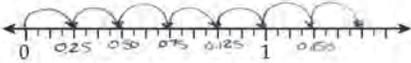
CHECK Check your answers by skip counting on the number line, and by using a calculator (e. g., press $0.2 + 0.2 =$; $+ 0.2 =$; $+ 0.2 =$; ...)

1. a. Skip by 0.2



b. Add on 0.2 using your calculator: 0.2, 0.4, 0.6, 0.8, 0.10, 0.12, 0.14

2. a. Skip by 0.25



b. Add on 0.25 using your calculator: 0.25, 0.50, 0.75, 0.125, 0.150



$$\begin{array}{r} 2.5 \\ + 3.6 \\ \hline 5.11 \end{array}$$

example

$70.5 \div 10 = 7\frac{1}{2}$

$6.9 \times 10 = 60.90$

D11 Does not line up the decimal points before adding or subtracting

The student adds or subtracts without considering place value, or starts at the right as with whole numbers.

example

Without using a calculator, can you see what is wrong with these answers? Explain what is wrong and correct them.

a. $0.43 + 0.6 = 0.49$

b. $1.15 - 0.6 = 1.9$

c. $50.1 + 6 = 50.7$

d. $25.06 - 5.1 = 9.45$

Handwritten work for problem a:

$$\begin{array}{r} 0.43 \\ + 0.6 \\ \hline 0.49 \end{array}$$
 She was correct

Handwritten work for problem b:

$$\begin{array}{r} 1.15 \\ - 0.6 \\ \hline 1.9 \end{array}$$
 She was correct

Handwritten work for problem c:

$$\begin{array}{r} 50.1 \\ + 6 \\ \hline 50.7 \end{array}$$
 She was correct

Handwritten work for problem d:

$$\begin{array}{r} 25.06 \\ - 5.1 \\ \hline 24.55 \end{array}$$
 She was wrong!

Handwritten work for problem c (alternative):

$$\begin{array}{r} 50 \ 1 \\ + \quad 6 \\ \hline 50.7 \end{array}$$
 She was correct

14 | MATHEMATICS NAVIGATOR

D13 Multiplies or divides incorrectly by a power of ten, or uses scientific notation incorrectly

example

$300 \times 10 = 30,000$

$$\begin{array}{r} 300 \\ \times 500 \\ \hline 00 \\ 0000 \\ + 15 \\ \hline 15000000 \end{array}$$

D15 Compares decimals by examining lengths—thinks that shorter lengths mean larger numbers (as with fraction denominators)

The student believes that two decimals can always be compared by looking at their “lengths,” or thinks that decimals with fewer digits are bigger because tenths are bigger than hundredths.

example

0.845 is smaller than 0.5 because 0.5 is tenths and 0.845 is thousandths.

Opening

Write these decimals in order from least to greatest.

0.75 0.4 0.375 0.25 0.125 0.04 0.8

0.4, 0.8, 0.04, 0.25, 0.75, 0.125, 0.375

D16 Compares decimals by examining lengths—thinks that longer lengths mean larger numbers (as in whole numbers)

The student thinks that decimals with more digits are larger because such decimals have more numbers; or, the student mistakenly applies what he or she knows about whole numbers.

example

Which is greater and why? 0.158 0.21

0.158 > 0.21 because 158 > 21.

1,234 is larger than 34 so 0.1234 is larger than 0.34.

2. Put these numbers in order from greatest to least.

0.8 0.05 0.5 0.15 0.465 0.55 0.75 0.4 1

1, 0.465, 0.75, 0.50, 0.15, 0.5, 0.4, 0.05

D17 Orders decimals based on the first or last digit of the number or the decimal part

example

Which is greater and why? 0.358 2.21

0.358 > 2.21 because 3 is greater than 2.

3. Now try to put these numbers in order from greatest to least, using your rules described in Problem 2.

34	3.15	3.85	33.4	200
378	379	340	3	40
3.05	3,005	300	3.7	37.25
4.01	3.9	3.09	2.9999	37.10

4.01, 40, 3.85, 379, 378, 37.25, 37.10,
 3.7, 3.4, 340, 3.15, 309, 3.9, 3.4,
 3,005, 3.05, 300, 3, 2.9999, 200

D21 Misapplies knowledge of fractions

example

0.204 > 0.240 because $\frac{1}{204} > \frac{1}{240}$

D26 Does not recognize the “denseness” of decimals, due to a lack of understanding of place value

The student thinks that there are no more decimal places, or only one decimal place, to the right of a given decimal; or, the student does not use zero as a placeholder when finding numbers between given decimals.

example

Write a number between 3.41 and 3.42.

There are no numbers between 3.41 and 3.42.

Write a number between 3.1 and 3.11

3.12

G3 Rounds incorrectly

example

Round to the nearest hundredth.

a. 10.502 *10.5*

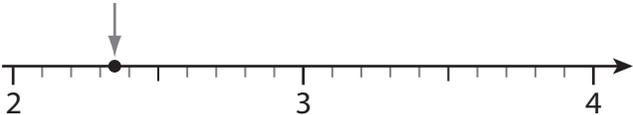
b. 14.007 *14.00*

NL1 Counts the number of tick marks as starting with 0 and ignores the range of the number line

The student incorrectly reads the decimals on a number line.

example

What number does the arrow point to?



0.35

NL2 Counts the number of tick marks instead of the number of intervals

The student incorrectly reads the decimals on a number line.

example

What number does the arrow point to?

3.35

Detailed description: A horizontal number line with arrows at both ends. Major tick marks are labeled 3, 4, and 5. There are 10 smaller tick marks between each major tick mark, representing hundredths. An arrow points to the 5th tick mark after 3, which is 3.35.

NL7 Fails to interpret the scale on a graph or number line

The student reads the marks on the ruler as whole numbers.

example

What number does the arrow point to?

10.5

Detailed description: A horizontal number line with arrows at both ends. Major tick marks are labeled 0, 1, and 2. There are 10 smaller tick marks between each major tick mark. An arrow points to the 5th tick mark after 1, which is 1.5. The student's incorrect answer, 10.5, is written to the right of the line.

O1 Does not use the correct operation

example

What number do you get if you subtract one-tenth from 13?

13.1

Detailed description: A text-based example box. The question asks for the result of subtracting one-tenth from 13. The student's incorrect answer, 13.1, is written below the question.

PV1 Does not understand or misinterprets the role of zero as a placeholder

The student misinterprets zeros when comparing and ordering numbers or finding numbers between given decimals; or, the student believes that zeros placed to the right of the decimal number change the value of the number.

 example	$1.5 = 1.05$ $0.4 > 0.400$ 0.81 is closer to 0.85 than 0.81 is to 0.8 .
---	---

Class Profile Instructions

About the Class Profile

Completing an analysis of student work gives you a clear picture of the strategies an individual student is applying to a particular problem or topic in mathematics. Such an analysis is even more powerful when it is applied to the Math Navigator class as a whole.

The Class Profile gives you both. By reading the Class Profile across a row, you can see where each student stands at any point in time. Reading down the columns allows you to see the strengths and needs of the entire class at a glance. By reviewing the Class Profile, you will be able to make decisions that target appropriate instruction to individuals, small groups, and the whole Math Navigator class.

The first pages of the Class Profile provide assessment items related to the content of the module. The last page is based on the mathematical practices from the Common Core State Standards for Mathematics.¹ On this page, record evidence of students using these practices.

Recording Data on the Class Profile

When you see—either through discussion, analysis of student work, or direct observation—that a student understands a concept, still has a misconception, or engages in a mathematical practice, make a note on your Class Profile. As the student's understanding increases, update the Class Profile.

Using the Class Profile

Review the Class Profile periodically during the lesson to help you decide which topics would be most beneficial for your students to focus on during the class discussion. Address topics that most of the students in the Math Navigator group need to learn during the show me, work time, or probing for understanding parts of the lesson. Address topics that only some students are struggling with during partner work or in conferences. If only one or two students need help with a topic, address the topic in an individual conference.

Give a copy of the completed Class Profile to each student's classroom teacher at the end of the module.

¹Common Core State Standards Initiative. 2010. "Common Core State Standards for Mathematics": 6–8. Accessed July 1, 2011. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

CLASS PROFILE (2 OF 3)

		Procedures				
Student Name	1					
	2					
	3					
	4					
	5					
	6					
	7					
	8					
	9					
	10					
		<i>P1: Correctly verbalizes decimals</i> <i>P2: Adds and subtracts decimals</i> <i>P3: Reads, writes, and compares decimals</i> <i>P4: Multiplies and divides by powers of ten</i> <i>P5: Multiplies and divides decimals</i>				
		Observed Errors				

Mathematical Practice Standards	
<p>MP1: Make sense of problems and persevere in solving them.</p> <p>MP2: Reason abstractly and quantitatively.</p> <p>MP3: Construct viable arguments and critique the reasoning of others.</p> <p>MP4: Model with mathematics.</p>	<p>MP5: Use appropriate tools strategically.</p> <p>MP6: Attend to precision.</p> <p>MP7: Look for and make use of structure.</p> <p>MP8: Look for and express regularity in repeated reasoning.</p>
Student Name	Observations
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



A Complete Solution to a Word Problem

includes all of the following ...



A written estimate



All work that you do



An equation (even if you solved it using column form)



A diagram, number line, table, or other representation



The answer to the question in a complete sentence



What to Do If You Get Stuck



Look at past work times



Look at the charts that are posted



Model the problem using counters or other materials



Sketch a diagram or other representation



Change the numbers to make the problem simpler



Write what you do know



Write down questions to ask later



Check other resources