# Program Philosophy

## The Guiding Principles of *Investigations 3*

*"Each time one prematurely teaches a child something he could have discovered for himself, that child is kept from inventing it and consequentially from understanding it completely."* —Jean Piaget (1970)

Do you agree with Piaget's 1970 words? Clearly, you cannot expect students to invent or discover mathematical ideas out of the blue. As teachers, you design experiences that guide students to the mathematical ideas that are important for them to understand. In the case of *Investigations 3,* that design work has already been done for you. This handout summarizes some of the commitments that *Investigations* has made to help students understand mathematics.

### Rethinking Traditional Instruction: Conceptual and Procedural Understanding

Think about how you were taught to multiply decimal numbers. Now think about the following multiplication problem:

$$10.024 \times 3.45$$

If you knew that the digits in the answer were 345828, how would you determine where you would place the decimal point?

To solve this problem, you might have remembered a rule about adding the digits after the decimal points and counting back from the right side. Since there are three digits to the right of the decimal point in 10.024 and two digits to the right of the decimal point in 3.45, that means the decimal point should be five positions from the right in the solution's digits—or 3.45828. If you thought about the problem this way, then you relied on **procedural understanding**.

If you relied more on predicting the size of the number and used methods of estimation, you might have thought that 10.024 is close to 10 and 3.45 is between 3 and 4, so the answer should be somewhere between 30 and 40—or 34.5828. If you thought about the problem this way, then you relied on **conceptual understanding**.

Why did these two approaches result in different answers?

When students rely on procedures that they do not understand, they often arrive at solutions that do not make sense. For example, a common misconception in fraction arithmetic is "adding across" numerators and denominators. After all, it works for multiplication. Students might arrive at a solution of $\frac{5}{10}$ when calculating $\frac{2}{4}+\frac{3}{6}$ and fail to realize that because $\frac{5}{10}$ is actually equal to each of the addends, the result must be larger.

So which type of understanding is more important? How do you balance them in traditional instruction?

As you might expect, both are very important. If you relied on the procedural rule described earlier, you probably realized at some point that the result did not make sense. You used conceptual understanding when you realized this. This is not to say that procedural understanding is without merit. Students who are weak in procedural understanding struggle with computational fluency. On one hand, they might understand the connections between $8 \times 8$ as a square with side lengths of 8 or two rectangles of $8 \times 4$, repeated addition of 8 eight times, and counting by eights, but procedural fluency is what will help them arrive at a solution quickly. On the other hand, students who are weak in conceptual understanding might have misconceptions such as the one in the fraction addition example. Rather than building on number sense and estimation, they might try to adapt "rules" they already know to new problems, which leads to nonsensical answers. This is why students need a strong grasp of both procedural and conceptual understanding.

*Investigations* is designed to strengthen both procedural and conceptual understanding through problem-solving situations, but typically students build conceptual understanding first. Students build procedural understanding through a variety of experiences to choose, apply, and practice efficient strategies. Your own students will design some of these strategies. When students use a combination of procedural and conceptual understanding, they are able to understand (and sometimes generate) elegant methods to solve problems.

## Illustration of Student Thinking

Consider the following student's reasoning about the problem 1,344 ÷ 21. In this example, the class has already solved 1,029 ÷ 21.

*Student: We already figured out that 1,029 was 49 × 21, so it's easy to know that 50 × 21 = 1,050. I need to get to 1,344, so I added 10 more 21s and got 1,260. I added by 21 and got 1,281; another 21 is 1,302. Then I'm 42 away from 1,344, so that's two more 21s. So let's see, that's 50, 60, 61, 62, 64.*

*Teacher: What do you mean, 50, 60, 61, 62, 64? Where did those numbers come from?*

*Student: I was just keeping track of how many 21s I had. First I had 50, then I add 10 more 21s. . .like that.*

Think about the evidence of procedural and conceptual understanding you see in this explanation. How would you record this student's thinking on the board in a way that honors the approach as described by the student? (Record your responses in the Program Philosophy Activity section in your Participant Guide.)

## The Guiding Principles

*Investigations 3* was developed with three guiding principles in mind. Read about each of these components on the following pages. Find examples of each guiding principle in your program materials.

The following information comes from TERC (2016):

Students have mathematical ideas.

Teachers are engaged in ongoing learning about mathematics content, pedagogy, and student learning.

Teachers collaborate with the students and curriculum materials to create the curriculum as enacted in the classroom.

1. *Students have mathematical ideas*. Students come to school with ideas about numbers, shapes, measurements, patterns, and data. If given the opportunity to learn in an environment that stresses making sense of mathematics, students build on the ideas they already have and learn about new mathematics they have never encountered. They learn mathematical content and develop fluency and skills that are well grounded in meaning. Students learn that they are capable of having mathematical ideas, applying what they know to new situations, and thinking and reasoning about unfamiliar problems.

2. *Teachers are engaged in ongoing learning about mathematics content, pedagogy, and student learning.* The curriculum provides material for professional development, to be used by teachers individually or in groups, that supports teachers' continued learning as they use the curriculum over several years. The *Investigations* curriculum materials are designed as much to be a dialogue with teachers as to be a core of content for students.

3. *Teachers collaborate with the students and curriculum materials to create the curriculum as enacted in the classroom.* The only way for a good curriculum to be used well is for teachers to be active participants in implementing it. Teachers use the curriculum to maintain a clear, focused, and coherent agenda for mathematics teaching. At the same time, they observe and listen carefully to students, try to understand how they are thinking, and make teaching decisions based on those observations.

As you become familiarized with how the guiding principles work in *Investigations 3*, you will begin to see that the common thread woven throughout the program is student thinking. Your students' ideas are the classroom currency that they use to make sense of mathematics. As you continue through this training, think about the ways that the program honors these principles consistently and coherently.

## References

Piaget, Jean. 1970. "Piaget's Theory." In *Carmichael's Manual of Child Psychology* Vol 1, edited by P. Mussen, 703–772. New York: John Wiley & Sons.